Real Options Approach in Petroleum Exploration and Production

Master Thesis

by

Dipl.-Ing. BSc. Oliver Hausberger

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Scope of Work

Dipl.-Ing. **Oliver HAUSBERGER** was requested to analyze in the master thesis at hand the topic:

„Real Options Approach in Petroleum Exploration and Production“

The main focus of this master thesis lies on the upstream sector of the oil and gas industry, commonly known as exploration and production. Upstream projects are typically high risk and capital intensive. To allocate resources in order to maximize the shareholder value, it is fundamental to accurately evaluate projects by considering market and technical uncertainties.

The commonly used traditional discounted cash flow analysis cannot adequately represent the real dynamic world. Therefore the real options approach, which represents an advancement of the net present value by taking the value of flexibility into consideration, will be the object of investigation of this thesis. The master thesis has to define the relevant terms and also give an overview of the different evaluation techniques used in the industry. Moreover the important steps and requirements of the process of resources allocation and the basic evaluation principles like the time value of money have to be described. A presentation and comparison of models and types of real options has to be included by giving special emphasis on the mathematical principles of financial and real options and their limitations.

The practical part of the thesis must deal with common real options used in exploration and production. Several exploration and production related examples have to demonstrate possible applications, including the calculation of the economic value based on the real options approach for each case. The results will then have to be analyzed and compared to the traditional discounted cash flow analysis.

Leoben, April 2013

o. Univ.Prof. Dr. Hubert Biedermann

Affidavit

I declare in lieu of oath, that I wrote this thesis and performed the associated research myself, using only literature cited in this volume.

Place/Date

(Oliver Hausberger)

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Notation

DCF	discounted cash flow
CF	cash flow
PV	present value
NPV	net present value
RO	real options
IROR	internal rate of return
EURIBOR	Euro Interbank Offered Rate
E&P	exploration and production
EMH	efficient market hypothesis
RWH	random walk hypothesis
VoF	value of flexibility
VoI	value of information
MCS	Monte Carlo simulation
DTA	decision tree analysis
PDE	partial differential equations
BS	Black & Scholes
PCP	put call parity
WACC	weighted average cost of capital
VIP	value improving practices
CAPM	capital asset pricing model
DAX	Deutscher Aktienindex
ATX	Austria Traded Index
MAD	market asset disclaimer
et al.	et alteri or et alii
f.	the following page
ff.	following pages

Abstract

Oil and gas companies need to allocate resources in order to maximize their shareholder value. After analyzing the technical feasibility and determining the economic value, projects are ranked and only the most promising ones get funded. Hence, the process of evaluation is crucial and has to be representative.

Even if companies are aware of the dynamic and risky environment the discounted cash flow analysis is the most commonly used tool to evaluate the economics of projects in the industry. But in today's world these methods typically fail to represent the real economic value. As a consequence the real options approach was introduced by Stewart Myer in 1977. Real options use the mathematical principles of financial options to evaluate the value of flexibility. This new approach is not a replacement of traditional methods, it is rather a logical advancement and can be thought as an "addon" to the net present value method. Over the last decades numerous real option models are developed.

The objective of the master thesis was to define the input parameters, the models and types used in the real options approach to evaluate E&P related projects and compare the results with the traditional discounted cash flow analysis. This new technique is intended to incorporate not only the value of information but also the value of flexibility and should therefore be able to enhance the valuation process and improve the strategic decision making.

At first the thesis discusses some important real option input parameters and valuation techniques including the classical and recent sophisticated approaches and compares them to create a clear insight. Whereas the classical methods solely rely on market information, recent developments not only focus on market uncertainties but also on private uncertainties. After selecting a model the projects inherent flexibility is then estimated and the types of options are selected. The most common types in the E&P industry are the wait-to-invest, the termination, the temporarily shut in and the operational options.

The final chapter includes numerical examples from the petroleum industry and analyses the advantages and disadvantages of real options. Understanding the different methods and applying the appropriate approach can effectively improve the valuation.

Key Words: discounted cash flow, real option analysis, market asset disclaimer, Black and Scholes, integrated real option approach, dynamic economic value, risk free rate, value of flexibility, value of information, petroleum exploration and production, evaluation techniques, capital asset pricing

1 Introduction

One of the most fundamental questions in the daily business is how to allocate the given resources in order to maximize the shareholder value[1]. Alfred Rappaport argues that the shareholder value as the free cash flow calculated by taken the earnings before interest and taxes increased by depreciation plus amortization and subtracted by changes in the working capital as well as capital expenditures over a period of time is a suitable measure of the financial health of a company. In his book "Creating Shareholder Value" he introduces this well known and widely accepted concept as a measure of a company's success[2].

A public company listed on the stock exchange like OMV is pursuing a corporate strategy to increase the enterprise value as defined by the capitalized earning power of the equity. Instead of focusing only on the profit, the new target is the shareholder value. The simple rule states that if the rate of return exceeds the cost of capital the investment should be undertaken. But because there is only a limited amount of resources that can be allocated to the different kinds of available projects all sources of income need to be classified and ranked[3]. The main object of capital budgeting is to invest in the projects with values that exceed their costs. Some may even consider capital budgeting as an art rather than a technique[4].

In order to standardize the process of allocating resources the necessary steps and requirements must be defined. The evaluation methods like the discounted cash flow analysis (= DCF) are probably the most vital tools in the budgeting process. The process itself consists of four important steps. Definition of projects that are compulsory (e.g. legal aspects, commitments, ...), determination and ranking of all other projects according to their economic value as well as capital allocation to the projects of highest value (Fig. 1-1).

Fig. 1-1: Capital Budgeting and the processes involved

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[1] cf. Baecker et al. (2003), p. 16 transl.
[2] URL: http://hbr.org/2006/09/ten-ways-to-create-shareholder-value/ar/1 (09.02.2014)
[3] URL: http://wirtschaftslexikon.gabler.de/Archiv/9868/shareholder-value-v8.html (21.04.2013) transl.
[4] cf. Brennan et al. (1985), p.39 f.

Despite the budgeting process claims to be complete, consistent in all matters and well documented[5].

The evaluation methods that are responsible for the ranking in the budgeting process need to fulfill at least six requirements to reflect reality. These are:

- cash flows instead of common variables used in accounting
- capture uncertainties involved in realization of cash flows
- flexibility of the management to react to changing circumstances
- irreversibility of decisions (e.g. sunk costs)
- calculations are based on market values
- implementation as an iterative method with continuous improvement[6]

Even if decision makers are aware of the importance of the mentioned demands, they are not necessarily able to satisfy them. In most cases because of the given situation and the implications it might be possible to agree on a specific range of values or even take it as a constant.

Managers are familiar with the traditional accounting methods like the net present value or the internal rate of return but because of the fast changing nature of today's market a new perspective with a rapidly growing literature developed. One of this recent advancements are real options and this technique allows managers to improve the integration of the given six points in a more consistent way. However, if projects do not imply specific characteristics like flexibility or uncertainty time and effort do not justify the extended calculations. Consequently it is of predominant importance to first choose the appropriate evaluation technique to determine a projects economic value[7].

Most evaluation methods give under various circumstances accurate numbers. Even if numbers are easy to handle and a ranking of projects is performed in no time, decision makers need to be aware of the limitations. All used equations in economics are only valid if specific conditions are met. Hence, every single evaluation method has its justification.

Probably the most frequently used technique is the DCF which is the preferred method if a project only has limited flexibility and low uncertainty (Fig. 1-). The reason is the very simple calculation that can be even done with a pocket calculator. The only two parameters needed are the discount rate and the cash flows over time. The estimated future cash flows are basically discounted using an appropriate risk adjusted time invariant a priori fixed discount rate. The sum of the discounted cash flows represent the net present value (=NPV). If the NPV is greater than zero invest otherwise reject the project. The static nature of this basic calculation scheme neglects the value of flexibility if new information arrives over time. DCF methods seem to fall short of capturing key value drivers and have shown to provide insufficient incentives for managerial actions to conform to the long term interest of shareholders[8]. A great disadvantage of DCF is that investing at a later date, not investing in projects with a great chance to grow or taking into account the actual size of the investment is not considered and leads to wrong decisions. It has been shown that the DCF underestimates the project inherent strategic value and only accounts for the downside of the risk without considering the chance that goes with it[9]. Only if uncertainty stays the same or

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[5] cf. Hilpisch (2006), p. 28 ff. transl.
[6] on the basis of Baecker et al. (2003), p. 22 transl.
[7] cf. Busby et al. (1997), p. 169 ff.
[8] URL: http://hbr.org/1982/05/managing-as-if-tomorrow-mattered/ar/1 (24.04.2013)
[9] cf. Hommel (1999), p. 121 transl.

there is no uncertainty at all the net present value (=NPV) as the most popular DCF method gives reasonable numbers.

However, in today's business uncertainty and flexibility might be high and changing over time when new information becomes available. So the question rises if any appropriate tool to satisfy these requirements exists?! And if such a method is available what are the limitations?

The possibility to react to the market conditions and recent information often creates additional value that needs to be captured by the analysis. Introduced by Stewart Myers in 1977 an alternative to DCF, with roots in the financial market, was therefore developed. He stated that the equilibrium market value of a company is the sum of real assets already in place and the value of future opportunities which he named "real options"[10]. This term today is known as a principle that can quantify the value of flexibility (=VoF) and the value of information (= VoI)[11].

The value of flexibility is exceptionally high if it is impossible to reduce the uncertainty with information and in the case of residual uncertainty. VoF creates value because single decisions may be split into multiple consecutive decisions over time with the opportunity to learn between the decisions and the option to react and respond accordingly.

The value of information on the other hand can in case of perfect information remove all uncertainty of an outcome by acquiring additional data. In real world business information usually only decreases and not eliminates uncertainty (= value of imperfect information). Furthermore if the new data is inconsistent with previous models, the VoI can even be negative[12].

A combination of the presented principles VoI and VoF as well as information about the market are key elements behind the real options (=RO) approach[13].

Moreover, the theory employs an intuitive analogy between financial options and real decision making flexibilities. But in contrast to financial options, real options are based on real assets like estates, projects or intellectual property. Instead of adding a premium to the discount rate to account for risks like in DCF, real options use risk neutral valuations and adjust the probabilities to reflect complete[14] and arbitrage free[15] financial markets. In principle projects can be interpreted as options if payoffs are subject to some form of risk and if management has certain degrees of freedom to allocate resources which will lead to sunk costs[16]. In other words RO are the right, but not the obligation, to invest a predetermined value in order to realize (or abandon) a project with an uncertain underlying value on or before a specified date[17].

So if the project under consideration has a high uncertainty, a high flexibility and at least some irreversible costs the real options approach is the preferred evaluation technique[18] (Fig. 1-).

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[10] cf. Myers (1977), p.4
[11] cf. Begg et al. (2002), p. 1 f.
[12] URL: http://msl1.mit.edu/MIB/rothstuff/Lecture4_InfoVal.ppt.pdf (18.02.2014)
[13] cf. Bratvold et al. (2010), p. 12 ff.
[14] any cash flow can be traded
[15] no price differences of the same good on two or more markets
[16] cf. Kodukula (2006), p. 5 f.
[17] cf. Hilpisch (2006), p. 32 f. transl.
[18] cf. Dixit (1994), p. 6 ff.

In comparison to the traditional discounted cash flow real options offer the possibility imply chances as well as risks in a more accurate way. Fig. 1-2 illustrates the value of flexibility as well as probability distribution of the outcome that has shifted to the right. Real options correctly consider risks and the possibility of upside potentials[19].

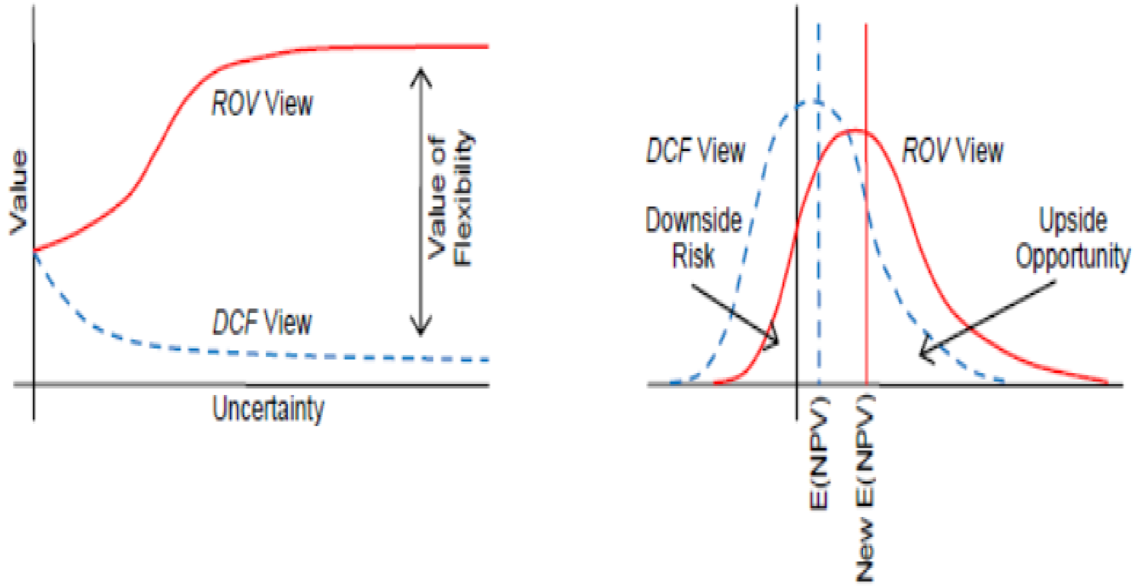


Fig. 1-2: Impact of uncertainty in value creation[20].

Hence, real options (=RO), as a combination of financial options and budgeting of real assets, seem to be a suitable method in today`s business environment. Still the complex mathematics can have a daunting effect on decision makers. To get approval the only way is to simplify the equations and make the calculations visible (e.g. in a diagram). The black box needs to be transformed into an easy process chart. However, all available information should be incorporated and nothing should be unconsidered.

The evaluation by real options is basically divided into three main parts. First of all one must determine if the real options approach is suitable for the given situation. This can be done by simply estimating the inherent projects flexibility and uncertainty. If these values are considerable high the incorporated types of options need to be set up. Only in rare occasions plain vanilla options reflect the given situation. Hence, more complex options need to be evolved in such a way that the key parameters and their interdependence in the system are correctly implemented. The final step is the calculation of the option value itself which is not trivial (Fig. 1-3). A good mathematical and methodological background and the correct determination of the input parameters are crucial for a proper evaluation of the project. Especially because of the way more complex mathematics this method often is a black box for the management. On the other hand well implemented real options don`t only give an enhanced project value, they also define the perfect strategy to achieve the highest possible free cash flow as an important performance indicator for shareholders (see shareholder value concept). But if the decision makers don’t trust any of these figures they won’t act accordingly and real options are meaningless[21].

[19] cf. Jafarizadeh (2009), p. 1 ff.
[20] cf. Jafarizadeh (2009), p. 3
[21] cf. Kodukula (2006), p. 60 ff.

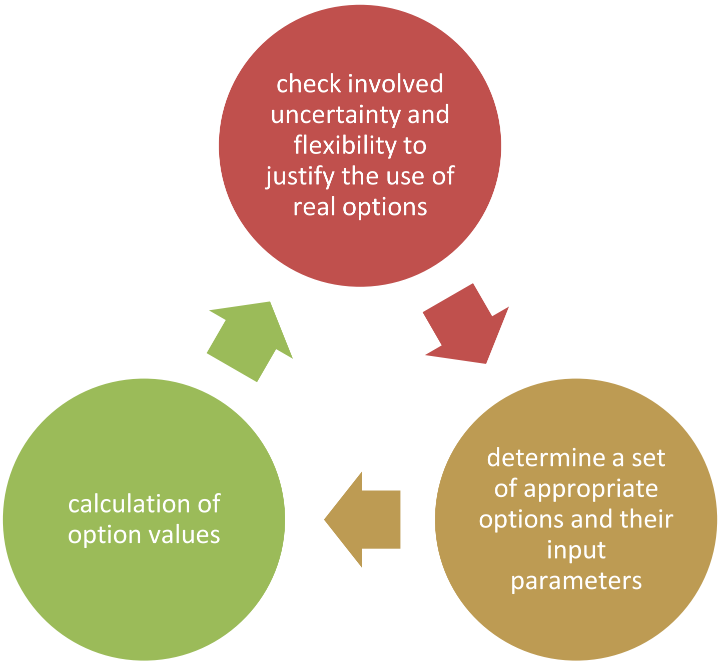


Fig. 1-3: Basic process of the real options approach

Doubtless the real options approach is definitely superior to any other evaluation technique. Some RO even give management the possibility to proactively change some input parameters in favor of an investment[22]. But real options are not a substitute for traditional methods but a logical advancement. Although the advantages of the RO are overwhelming it is not widely accepted by the industry. The biggest challenge lies in the required knowledge and its operational practicability[23].

Other quite common tools are sensitivity and scenario analyses as well as Monte Carlo simulations. These methods are particularly useful if the uncertainty is in contrast to the flexibility high. But these approaches also require that the appropriate policy is known in advance[24]. Even if uncertainties of single parameters can be adequately characterized in the sensitivity analysis, their interdependence will not be considered[25]. This can lead to wrong decisions and loss of information. Therefore scenario analyses are often performed where all value drivers at the same time are changed and a best case, worst case and base case scenario are compared to each other.

The Monte Carlo simulation (= MCS) is actually an enhancement of the traditional methods in such a way that every input parameter is defined by an appropriate distribution. During the simulation many thousands of random numbers within the range of the input values will be generated and the distribution of the output calculated. This method can give a quite good insight into complex risk structures. Nevertheless flexibility is only partially covered by Monte Carlo simulations.

Decision Trees (= DTA) are another evaluation method for a clear representation of uncertainty and flexibility. This effective technique to evaluate and demonstrate contingent

[22] cf. Hilpisch (2006), p. 211 ff. transl.
[23] cf. Hommel et al. (1999), p. 122 f. transl.
[24] cf. Brennan et al. (1985), p. 41 f.
[25] also called ceteris paribus which is a Latin phrase for „other parameters stay the same“

decisions as a road map is well known in the industry. The project value according to a decision tree is defined as the combined expected values of each outcome at a given node. The big challenge is despite the unknown and likely biased probabilities the static discount rate which does not change over time even if the risks vary[26].

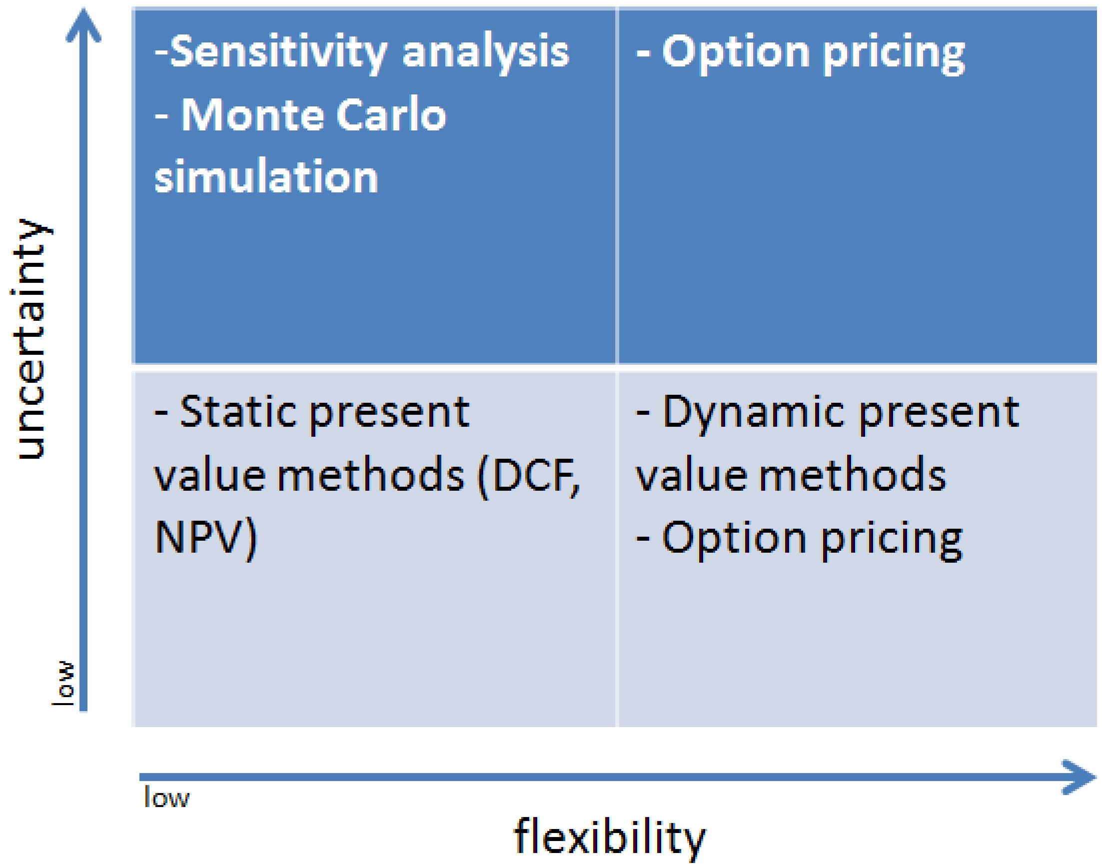


Fig. 1-4: Evaluation methods and their field of application[27].

OMV would like to benefit from new evaluation methods like real options to be able to improve the budgeting process. The major focus should be put on the practical usage instead of theoretical black box models. Various risks and the flexibility of the management must be considered. RO on the other hand can never be a substitute for DCF[28]. Therefore it is important to evaluate projects with all available methods in order to compare the results and state reasons for their differences.

One dissertation and one master thesis were already written under the supervision of the Mining University of Leoben and OMV upon real options pricing and the real options approach. Both papers give an excellent overview and the dissertation also developed an advanced real options evaluation tool. The thesis at hand will concentrate more on the simplification of the mathematics and redesign the in the recent literature described models with the intention of high acceptance. In the authors opinion it does not make any sense to

[26] cf. Hommel et al. (1999), p. 127 f. transl.
[27] cf. Hommel et al. (1999), p. 129 transl.
[28] cf. Kodukula (2006), p. 10 f.

develop a complex model that cannot be understood by decision makers within decent time.

After an introduction to the mathematics and a description of the traditional evaluation tools as well as the definition of real options the emphasis lies on the calculation of different flexibilities within real oil and gas projects of OMV. Because real options are based on market values the extractive industry has the great advantage that prices, like the oil price, are easier to observe than for example new software developments. Another simplification like in financial options is that if you own the license to extract any kind of raw materials from a specific area it is the company's exclusive right for a certain period of time.

Many different models and types of options will be considered and the results will be compared and interpreted. Moreover, the in this thesis described models are not calculated with various available tools on the internet because the emphasis lies on gaining knowledge and not typing in numbers and hitting the "calculation" button. The most neglected consideration is according to the principle "garbage in – garbage out" the influence and the determination of input parameters. A computer will unquestioningly process every input parameter and produce useless output which will lead to wrong decisions.

The last chapters review and discuss the calculated results and gives further suggestions for OMV.

2 Fundamental Terms and the Definition of Real Options

2.1 Definition of Fundamental Terms and Mathematical Background

This chapter aims to define the key terms later used in the evaluation process. It is also meant to give an overview of the basic equations used in economics.

2.1.1 Project Choice

In order to decide which projects should be considered in the decision making process three main factors are important.

First and foremost is the profitability of the project. Companies define various economic criteria that need to be achieved. For example the internal rate of return (= IROR) or the net present value (= NPV). However, even if the NPV is negative some projects with reasonable high chances of additional success can get funded. Managers should never make decisions solely based on economic indicators but consider data from geosciences and engineering as well.

Another factor is the competition against other projects. Only a limited budget is available and the decision makers need to select those projects that fit the firm's growth, strategy and commitment.

The third and last factor is the corporate strategy. Some projects have a greater strategic value than pure financial numbers may indicate. This could be employee morale or expertise within the organization[29].

2.1.2 Project Cash Flows

Usually calculated for each year of a project life, the cash flow represents the inflows and outflows over a certain period of time. The basic outflows are investment costs for field development or abandonment, whereas net revenues in the production phase are typical inflows[30]. The sum of the net revenues, the investments, the operating costs and taxes give the net cash flow (= CF)[31].

$$(1)\ CF = (Revenue - Royalties) - Investment - OPEX - Tax$$

If all the yearly CFs of a project are discounted with an appropriate interest rate to the present and added up the result is the net present value. The determination of the cash flows is the most fundamental step in valuing the projects profitability.

A typical exploration and production (= E&P) project in the oil & gas industry can be divided into three main stages. The first one is the pre project phase consisting of the potential identification and definition of the reservoir as the primary technical task of geologists and geophysicists followed by the appraisal phase in which the first drilling and testing operations are executed. Because during this stage there is little or no production of petroleum at all, it will be characterized by a negative cash flow. However, once production of the

[29] cf. Hutta et al. (2002), p. 9 f.

[30] cf. Kodukula et al. (2006), p. 15

[31] cf. Hutta et al. (2002), p. 20 f.

hydrocarbons in the second stage starts the net cash flows will turn to positive values. The final stage is the abandonment where the well abandonment and the recultivation of the environment takes place. The cash flows in the third stage will again be negative (Fig. 2-1)[32].

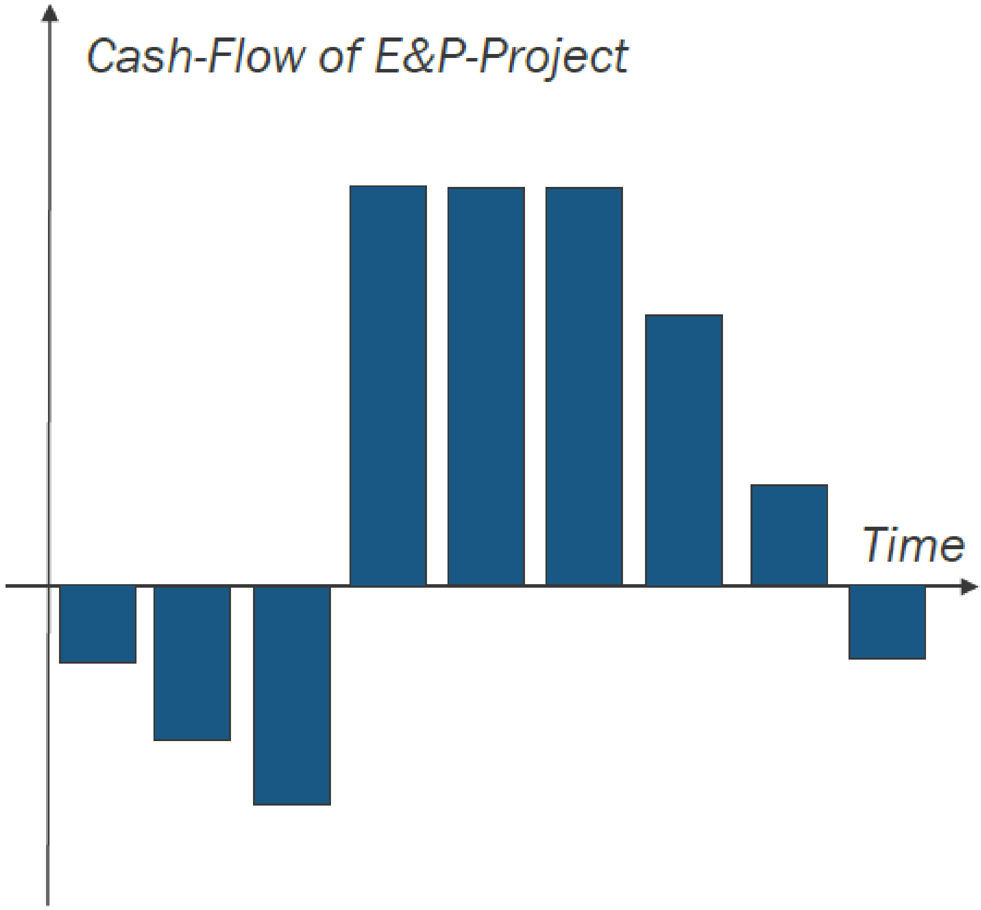


Fig. 2-1: Cash flow of a typical E&P project[33].

Another essential key figure is the maximum exposure. Since management needs to approve the budget it is important to know the value at which the expenses minus the revenues are at a maximum or in other words where the cumulative cash flow is at its lowest. The purpose is to give management an idea about the maximum money at risk if the project has to be stopped for some reason. Typically this point is right at the end of the exploration phase before a significant production of hydrocarbons took place[34].

Besides the maximum exposure also timing and the size of the investment are important. Smaller reservoirs can as well generate large cash flows. Timing is critical because companies aim for fast returns that will influence the NPV in a very positive way.

[32] cf. Hausberger, Högn, Soliman (2012), p. 5 f.
[33] source: Theodoridou (2012), p. 7
[34] cf. Hutta et al. (2002), p. 23 f.

2.1.3 Time Value and Interest Rate

"A dollar today is worth more than a dollar tomorrow!" is a well known idiom. Consequently time has a great impact on the economics and to consider the effect of time economists use an interest rate. If the future value of present money is calculated the procedure is called compounding otherwise its discounting.

In addition to that inflation decreases the value of money. To distinguish these two effects the terms real and nominal are used. Real values refer to the economic values that are adjusted for inflation, whereas nominal values represent the value in a certain year. If the index (e.g. GDP deflator or consumer price index) is known one can transform real to nominal values.

$$(2)\ \frac{nominal\ value}{real\ value} * 100 = Index$$

In real option models the risk free annual interest rate instead of any adjusted interest rate is used. The risk free rate is usually determined on the basis of the U.S. Treasury spot rate of return[35]. Government bonds are issued by a national government to an investor who is willing to loan a certain amount of money, for a certain period of time with a certain interest rate to the country. The credit risk is generally quite low because the government can simply raise taxes or the central bank creates additional value to be able to redeem the bond. Therefore the government bonds are referred to as risk free bonds with a risk free interest rate. Rare occasions like the financial crises in Russia 1998 exist where a government went bankrupt and could not pay back their bonds[36]. An indication about the credit risk of a country is the rating of the so called credit rating agencies which again affects the interest rate. The three biggest rating agencies are Standard & Poor`s, Moody`s Investors Service and Fitch Ratings.

The risk free interest rate is one of the most important input parameters of real options and should be according to the literature determined using U.S. Treasury or equivalent European bonds with a similar time to maturity[37]. By definition it is the rate of return of a riskless investment only including the default risk. Besides the U.S. Treacheries the Euro Interbank Offered Rate is of great significance for European projects. Fig. 2-2 and Fig. 2-3 show the U.S. and EURIBOR interest rates of May 2013, which will be further used for the calculations in the thesis at hand.

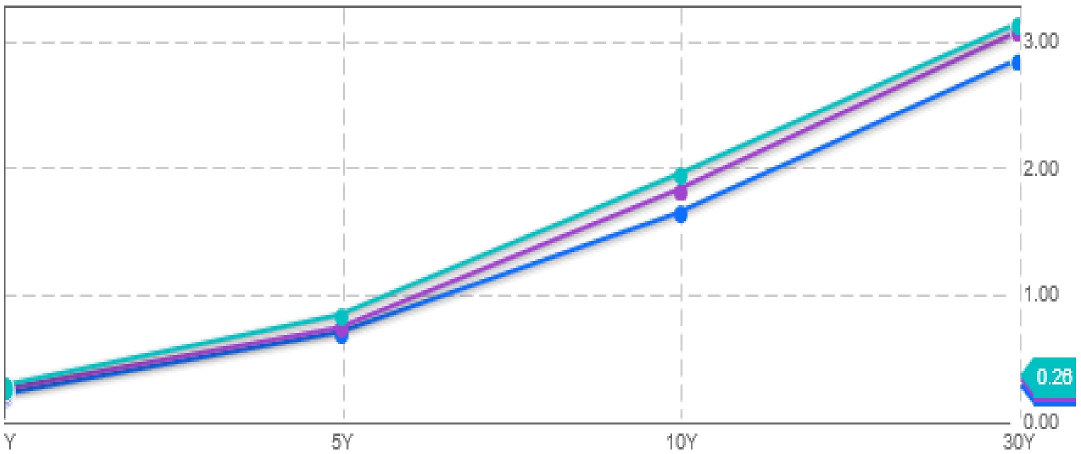
[35] cf. Kodukula et al. (2006), p. 94

[36] URL: http://www.bpb.de/apuz/25451/die-finanzkrise-in-russland-im-gefolge-der-asienkrise?p=all (02.05.2013), transl.

[37] URL: http://www.investopedia.com/financial-edge/0811/are-u.s.-bonds-really-risk-free.aspx (02.05.2013)

	05-03-2013	05-02-2013	05-01-2013	04-30-2013	04-29-2013
Euribor - 1 week	0.083%	0.083%	0.083%	0.083%	0.083%
Euribor - 2 weeks	0.092%	0.092%	0.092%	0.092%	0.091%
Euribor - 3 weeks	0.100%	0.101%	0.101%	0.101%	0.100%
Euribor - 1 month	0.113%	0.116%	0.116%	0.116%	0.117%
Euribor - 2 months	0.159%	0.165%	0.165%	0.165%	0.166%
Euribor - 3 months	0.201%	0.207%	0.207%	0.207%	0.207%
Euribor - 4 months	0.236%	0.242%	0.242%	0.242%	0.242%
Euribor - 5 months	0.272%	0.281%	0.281%	0.281%	0.282%
Euribor - 6 months	0.302%	0.313%	0.315%	0.315%	0.317%
Euribor - 7 months	0.333%	0.345%	0.346%	0.346%	0.347%
Euribor - 8 months	0.363%	0.377%	0.376%	0.376%	0.378%
Euribor - 9 months	0.398%	0.411%	0.411%	0.411%	0.413%
Euribor - 10 months	0.428%	0.442%	0.442%	0.442%	0.445%
Euribor - 11 months	0.463%	0.478%	0.479%	0.479%	0.480%
Euribor - 12 months	0.493%	0.510%	0.510%	0.510%	0.513%

Fig. 2-2: Euribor interest rates at specific dates[38].



Name	Yield	1 Day	1 Month	1 Year	Time
US Treasury 2 Year Yield	0.20%	+1	-2	-5	08:30:00
US Treasury 5 Year Yield	0.69%	+4	-4	-14	08:30:00
US Treasury 10 Year Yield	1.63%	+1	-13	-25	08:27:00
US Treasury 30 Year Yield	2.83%	+1	-16	-23	08:22:00

Fig. 2-3: U.S. Treasury Yields for 2, 5, 10 and 30 years[39].

[38] source: http://www.euribor-rates.eu/current-euribor-rates.asp (03.05.2013)

[39] source: http://www.bloomberg.com/markets/rates-bonds/government-bonds/us/ (03.05.2013)

The risk free interest rate can be seen as refinancing costs of the investment. If a company invests in a project they spend money which they must refinance e.g. at the capital market through bonds. The least interest expenses are those of government bonds because there is no credit risk involved. In case of an option to expand as one type of real options an increasing interest rate results in a greater RO value, whereas in options to contract this can be the way around[40]. So the effect of the interest rate on real options heavily depends on the type of the executed option.

The most common RO models use instead of a discrete interest rate a continuous rate. With a discrete rate the interest is only added at discrete time intervals e.g. per year while a continuous rate can be applied every time. Mathematically this can be described using the following formula (3).

$$(3)\ K_t = K_0 * (1 + \frac{r}{m})^{m*t}$$

K_t represents the value after the period t, K_0 the investment, m the time of the interest premium and r the nominal risk free interest rate. If $m \rightarrow \infty$ the interest will be added every single time and the outcome is the Euler`s constant e ~ 2,7182 (4).

$$(4)\ \lim_{x \rightarrow \infty} (1 + \frac{1}{m})^m = e$$

Substitution of (4) in (3) yields (5).

$$(5)\ K_t = K_0 * e^{i*t}$$

In case of the discrete interest rate K_t can be calculated using equation (6).

$$(6)\ K_t = K_0 * (1 + r_d)^t$$

Now if both have the same result, the corresponding continuous rate can be determined (7)[41].

$$(7) r = \ln{(1 + r_d)}$$

The following example should illustrate the difference in the interest rates. A company invests $80 million in a project over ten years. Because the management would like to evaluate the project using the RO approach, in a first step the refinancing costs need to be calculated. The annual ten year risk free U.S. Treasury yield is 1.63% (see Fig. 2-3). With (7) the equivalent continuously interest rate is 1.617%. Even if this is less than the 1.63% the final value after ten years is the same. Additionally the refinancing costs are 94.04 – 80 = $14.04 million.

Besides the risk free rate especially for the discounted cash flow analysis the so called weighted average cost of capital (= WACC)are a significant parameter. In most cases a project will be financed by equity and debt. Consequently the interest rate at which the cash flows will be discounted needs to be weighted according to the source. For debt this is an easy process because all vital parameters like accounting value or interest rate can be directly found on the balance sheet in the annual report. In contrast the interest rate at which the equity needs to be scaled is based on the market. According to the current risk free rate, the risk structure of the organization as well as the market capitalization, the interest rate for

[40] cf. Hilpisch (2006), p. 96 transl.
[41] cf. Tietze (2010), p. 89 ff. transl.

equity can be estimated. The WACC is one of the most important methods in the capital asset pricing model (= CAPM)[42].

2.1.4 The Weighted Average Cost of Capital

Because an organization is financed by a combination of debt and equity a weighted average is used in the discounted cash flow method. Of course if a project is exclusively financed by equity the WACC will only consider equity.

In general the first step according to the capital asset pricing model (= CAPM) the costs of equity needs to be determined. Equity Shareholders expect some return on their investment and this return is considering being costs for the company. Moreover, investors should not only be compensated for the time value of money but also for the incorporated risk. So the expected return in the CAPM is the sum of the risk free rate as well as the product of beta and the difference between the market return and the risk free rate (8).

$$(8)\ r_a = r_f + \beta * (r_m - r_f)$$

where r_a is the expected return, r_f the risk free rate and r_m the expected market return. The beta defines the volatility of the underlying security. The overall market has a beta value of 1 and if a security is more volatile than the market it possesses a beta greater than 1. On the contrary securities that are less volatile than the market have a beta less than 1 and therefore are considered to have lower returns. A security with a beta of zero would have no systematic risk. So in other words

Because for projects no beta values are available, a publicly traded security, a so called twin security which has the same risk profile or in other words the same cash flow profile is used as a proxy.

Several webpages like "http://markets.ft.com" offer the possibility to search for companies and obtain their beta values.

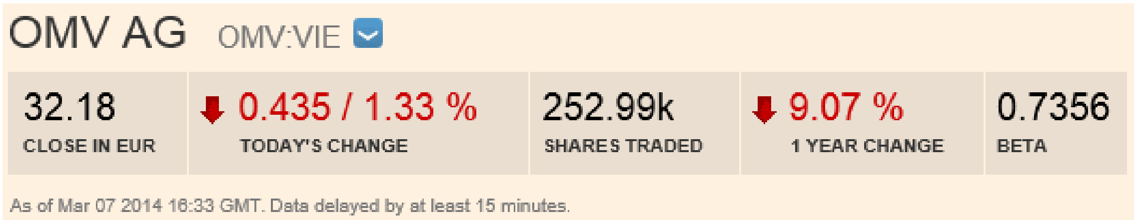


Fig. 2-4: OMV AG on the webpage http://markets.ft.com[43].

Let's consider another example that can easily explain the beta value and the incorporated determination. The DAX (=Deutscher Aktienindex) as the most important blue chip stock market index in Germany rises by 2% and the security under consideration by 4%. The beta can be calculated as the gradient and is therefore 2.

So in other words a market portfolio is created that has per definition a beta of 1. Usually for the market portfolio the typical country indices are used (e.g. DAX, ATX). The beta value corresponds to relative changes of market portfolio and share prices.

The expected market return as the third unknown variable in equation (8) on the other hand is often quite difficult to obtain. Usually the average return of the market over a spe-

[42] cf. Schacht & Fackler (2009), p. 237 ff. transl.

[43] URL: http://markets.ft.com (03.03.2014)

cific period of time which should be equal to the time for the determination of the risk free rate is used.

As an example the market return between 1991 and 2007 of the ATX (=Austria Traded Index) is presented in figure 2-11.

year	yield	year	yield	year	yield
1991	−11,7	1997	13,6	2003	34,4
1992	−15,3	1998	−13,5	2004	57,4
1993	51,0	1999	6,9	2005	50,8
1994	−6,5	2000	−10,4	2006	21,7
1995	−9,0	2001	6,2	2007	1,1
1996	18,8	2002	0,8		

Fig. 2-5: The market return of the ATX from 1991 to 2007[44].

The second part of the WACC are the costs of debt which can usually be taken directly from the annual report. If the interest rates of the debt increase over time an average should be used instead. Because the costs of debt are tax deductible the costs after taxes should be applied.

The equity and debt ratio is as well needed to calculate the WACC. Generally the market capitalization as the equity ratio should be estimated according to equation (9). Both ratios represent market based values instead of the classical financial approach (compare equity-to-assets ratio).

$$(9)\ \text{equity ratio} = \text{share price} * \text{number of shares}$$

E.g. OMV AG has about 327 million shares on the market and with the share price of 32.18 (Fig. 2-4) the market capitalization would be about $ 10.5 billion.

The debt ratio is simply the accounting value from the annual report as a percentage of the total capital.

Finally the rate of taxes “rt” is taken from the balance sheet in put in equation (10) to calculate the WACC.

$$(10)\ \text{WACC} = \text{equity ratio} * \text{ra} + \text{debt ratio} * \text{ra} * (1 - \text{rt})$$

[44] cf. Alt (2013), p. 39 ff. transl.

2.1.5 Present Value

Economists now combine the two already described principles of project cash flows as well as the time value of money and discount all future cash flows over the entire life of the project to the present day. Even if this procedure is a very basic and simple one it also applies to more sophisticated techniques like RO.

The present value (= PV) of future cash flows can be calculated using equation (8). The term $1/(1+r_d)^t$ is called the discount factor. In case of a continuous discount rate instead of a discrete equation (9) is the appropriate one[45].

$$(8)\ PV = \sum_{t=1}^{t} \frac{CF_t}{(1 + r_d)^t}$$

$$(9)\ \mathrm{PV} = \sum_{\mathrm{t}=1}^{\mathrm{t}} \mathrm{CF_t} * \mathrm{e}^{-\mathrm{r}*\mathrm{t}}$$

In the most basic evaluation method you simply add up all the discounted free cash flows. The great disadvantage is that all input parameters are static and don't vary in time. Nevertheless this discounted cash flow analysis called net present value is one of the most applied techniques. If the NPV according to equation (10) is negative the project is considered to be financially unattractive. Because in a portfolio projects compete against each other, projects with a positive NPV may not get funded.

$$(10)\ NPV = \sum_{t=1}^{t} \frac{CF_t}{(1 + r_d)^t} - \sum_{t*=1}^{t*} \frac{Investment_{t*}}{(1 + r_d)^{t*}}$$

Usually a sensitivity analysis is performed after the calculation of the NPV to determine the critical input parameters. This is done by simply changing one parameter at a time and examine the influence on the result. Moreover, a scenario analysis can be carried out to define the worst and the best case.

Despite the disadvantages the NPV still allows the analyst to get a first impression of a projects economics. Nevertheless a decision should never be based on the NPV alone. Especially the RO approach can give a very detailed insight and can quantify possible decision over the entire life of the project. Instead of the NPV as a now or never decision, RO methods illustrate the value of information and flexibility at any time. However, the basic underlying principles are similar for both methods even if fewer assumptions must be made and more information can be incorporated in the real options approach[46].

[45] cf. Kodukula et al. (2006), p. 16 f.
[46] cf. Kodukula (2006), p. 10 f.

2.1.6 Payout

Two often used yardsticks are payout and payback. Payout is the point in time when the cumulative cash inflow equals the cumulative cash outflows[47]. Managers always seek projects with the shortest possible payout time. Because of its simplicity these two figures were one of the first popular performance indicators in the oil and gas business.

Yardsticks of Petroleum Economics	
Before 1930	Payout / Payback
1930 – 1942	Profit / Investment
1943 – 1947	War / Post War Years
1948 – 1958	Profit / Investment
1959 – Present	Time Value of Money

Fig. 2-6: The yardsticks of petroleum economics[48].

2.1.7 Return on Investment and the Profitability Index

The return on investment is simply the undiscounted benefit of an investment divided by the cost of the investment itself. Usually expressed as a percentage its advantage is that it recognizes a profit in relation to the size of an investment. In contrast to the ROI the profitability index (= PI) is calculated as the ratio of the discounted present value and the initial investment and is therefore a more realistic measure[49]. A ratio lower than 1.0 indicates that the initial investment is higher than the present value of the project, hence the financial attractiveness is very low and it is exceptionally unlikely that the project will be carried out unless it has some high strategic value for the company or is part of a commitment. The profitability index is usually used to rank projects according to their value created per unit of investment.

The following example should illustrate the application in the corporate finance. Let's consider a late life field opportunity. Water infill in a mature onshore field is estimated to increase production and generate an additional cash flow according to the following figure (Table 2-1).

Table 2-1: Profitability index of water infill project.

Year	Costs	Net Revenue	Cash Flow @ 10%	Cum. Cash Flow
0	-5000000		-5000000	-5000000
1		2500000	2272727	-2727273
2		1700000	1404959	-1322314
3		900000	676183	-646131
4		600000	409808	-236323
5		500000	310461	**74138**

[47] cf. Hutta (2002), p. 22 f.
[48] cf. Megill (1979), p. 12
[49] cf. Kodukula (2006), p. 15 f.

The investment costs due to drilling additional wells and the needed equipment are expected to be around $ 5 MM. Management would like to know the net present value @ 10% interest rate as well as the profitability index of that particular project. First of all the NPV needs to be calculated as the cumulative discounted cash flow. In a second step the present value will be divided through the initial investment and the calculated ratio be analysed. The NPV of the presented project is just with about 74000 just slightly above zero so a real options approach is recommended. Besides the profitability index is consequently $5.074 MM divided through $ 5 MM. Because the resulting profitability index is just marginally above 1.0 the financial attractiveness of this specific project is questionable and water infill as one of the late life opportunities might need to be reconsidered.

2.1.8 Internal Rate of Return

The internal rate of return (=IROR) is the discount rate at which the net present value as the sum of all discounted cash flows over the entire life of a project equals zero. In other words the project breaks even at the IROR. Even if cash flows at the end of a project lifecycle often only have an insignificant impact this figure gives managers a quite clear sign if an investment can potentially generate the needed return. Additionally it is frequently used to rank projects. The biggest disadvantage is that the IROR does not provide the decision maker with information about the size of the free cash flows[50].

2.1.9 Taxation and Government Regulations

Every economic analysis should also consider taxation and government regulations and possible changes over the projects lifetime. High taxes and royalties can turn the table and make profitable projects uneconomic[51].

2.1.10 Probabilistic Parameters and their Distributions

Real option values like the stock price are generated using random variables. A random variable is a function that assigns a real number to every possible outcome of an experiment. Again a discrete variable is a specific value whereas a continuous one can take any number within an interval. Accordingly a discrete probability function is a function where each outcome has a specific probability while a continuous function is only meaningful between two boundaries that define an area under the curve that is related to the probability (Fig. 2-7).

[50] source: Theodoridou (2012), p. 23 ff.
[51] cf. Hutta (2002), p. 7 f.

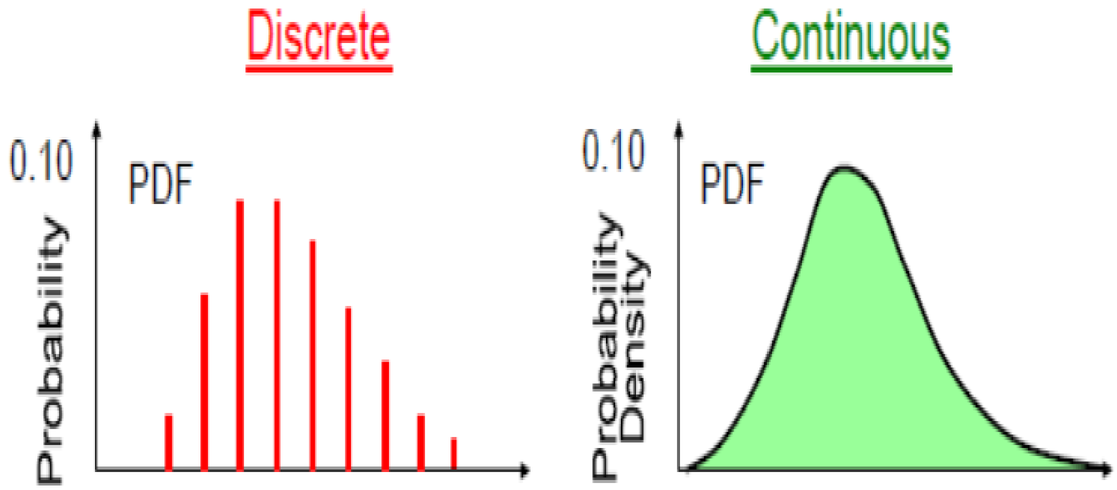


Fig. 2-7: A discrete probability density function on the left in comparison to a continuous probability density function on the right side[52].

Common discrete distributions in the E&P business are for example lognormal distributions for the variability of reserves in a field, Poisson distributions for the number of stuck pipes, machine breakdowns or fault intersections during a given time period as well as hyper-geometric distributions for the number of added slots on a platform or the number of wells to be drilling to meet contract throughput.

In case of a continuous random variable X the appropriate continuous probability density function f(x) can be defined as (11)

$$(11)\ f(x)dx = P(x - \frac{1}{2}dx \leq X \leq x + \frac{1}{2}dx)$$

if f(x) satisfies the following conditions

$$f(x) \geq 0$$
$$\int f(x)dx = 1$$

The cumulative distribution function F(x) on the other hand can be described using equation (12) and (13)[53].

$$(12)\ F(x) = P(-\infty \leq X \leq x)$$
$$(13)\ F(x) = \int_{-\infty}^{x} f(x)dx$$

In figure 2-7 the probability density function and the cumulative distribution function for a log-normal distribution are demonstrated. The cumulative distribution function can easily be recognized because it approaches one. This is because the function represents the likelihood of a random value to be smaller or equal to a chosen variable value. The probability density function on the other hand describes the likelihood of a random variable to take on a selected value. It is very similar to a discrete histogram and the peak is the so called mode as the most frequent value in the data set. As a rule of thumb about 95% of all possible

[52] cf. Beg (2012), p. 14
[53] cf. Veerarajan (2003), p. 37 f.

values are within the mode +/- two times the standard deviation of the function. Functions can also be bimodal or multimodal in case they have several local maxima (e.g. dual porosity systems in rocks). However, in economics most functions are unimodal or in other words only have one maxima.

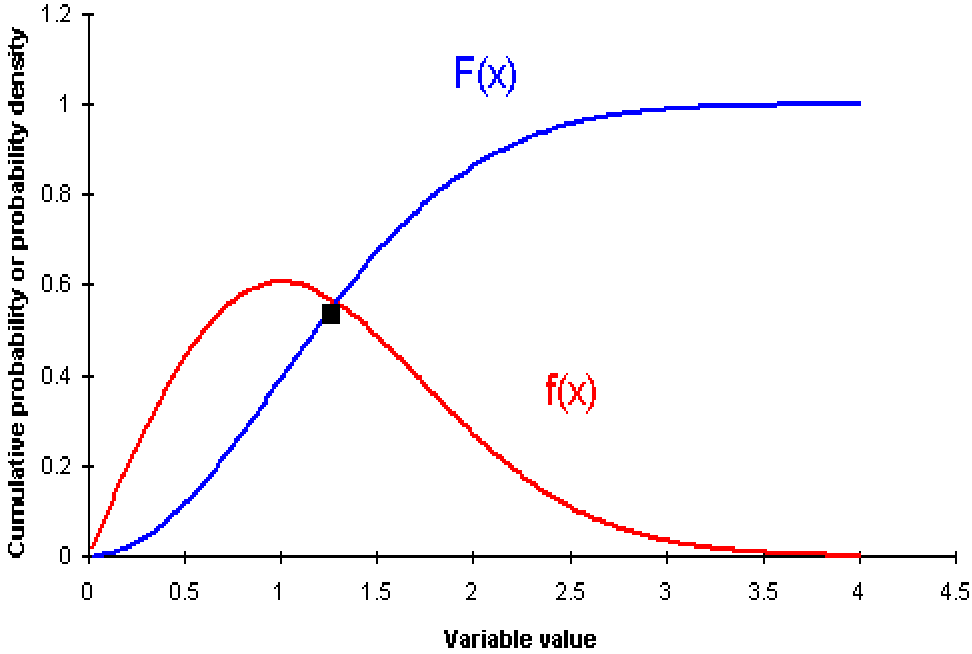


Fig. 2-8: A lognormal distribution presented as the probability density function f(x) and the corresponding cumulative distribution function F(x)[54].

In most cases it is easier to characterize a distribution by specific values or statistical averages instead of stating the actual function. Two of the most important figures are the expected value or mean μ (14) and variance σ^2 (15). Furthermore, the square root of the variance is the standard deviation[55].

$$(14)\ \mu = \int x * f(x)dx$$

$$(15)\ \sigma^2 = \int (x-\mu)^2 * f(x)dx$$

[54] URL: http://www.vosesoftware.com/ModelRiskHelp/index.htm#Probability_theory_and_statistics/The_basics/Probability_equations/Probability_density_function_(pdf).htm (22.02.2014)

[55] cf. Veerarajan (2003), p. 120 f.

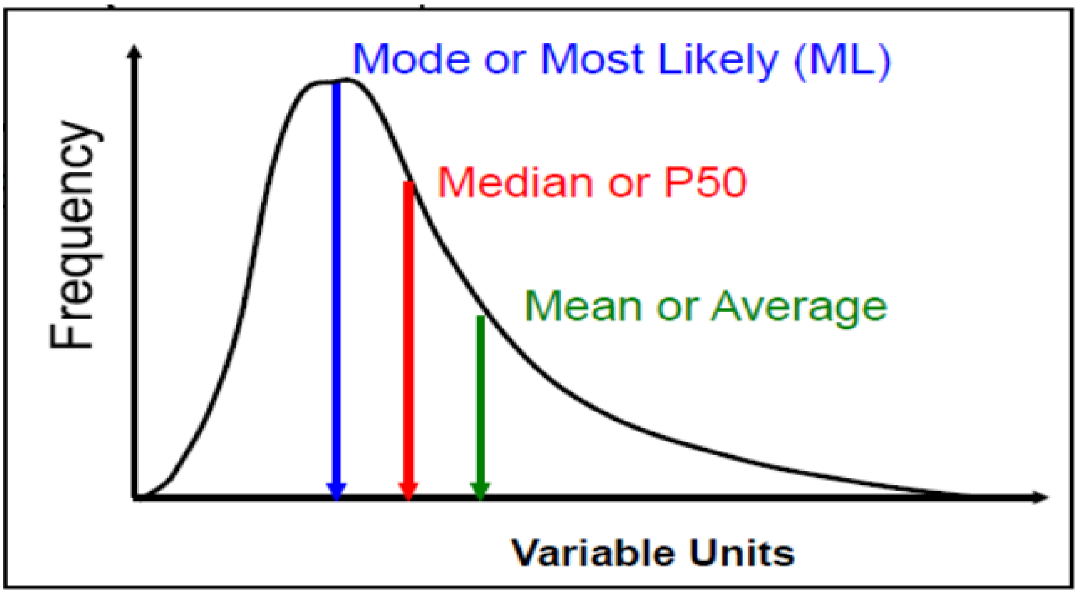


Fig. 2-9: Statistical figures of an asymmetrical distribution. Compared to a symmetrical distribution the median, the mean and the mode are not equal[56].

In financial mathematics the three commonly used functions are the normal or Gaussian, the lognormal and the triangle distribution. Fig. 2-10 shows the most important functions in economics.

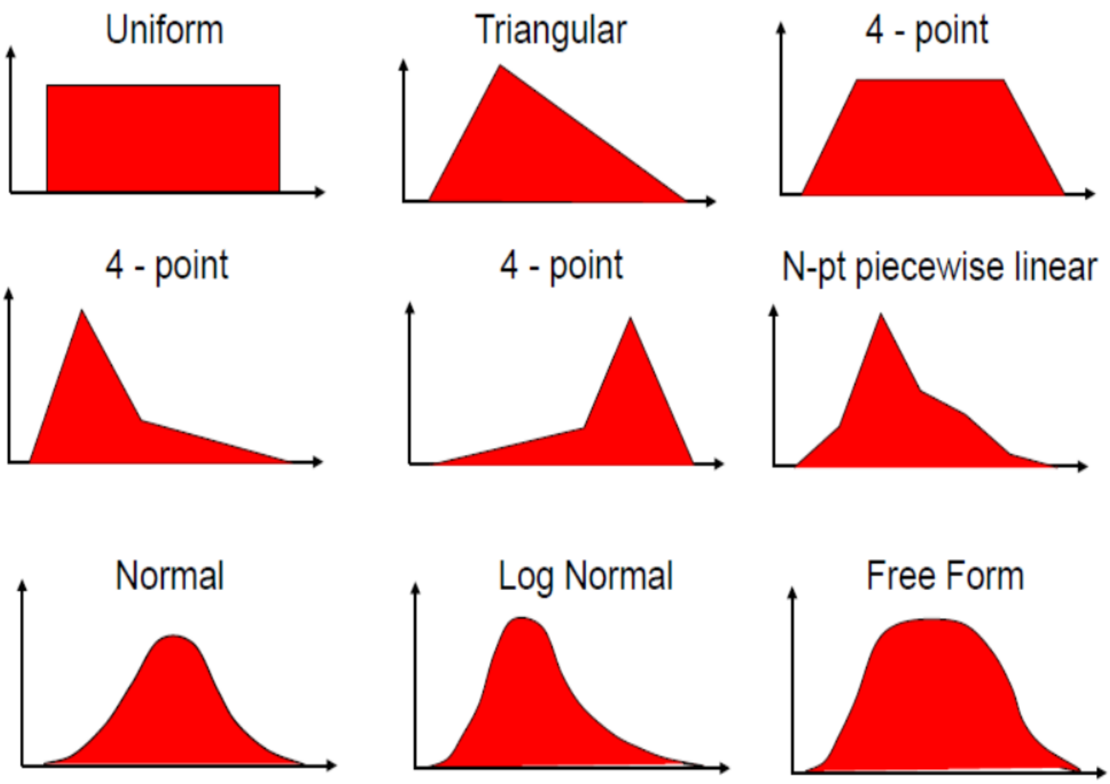


Fig. 2-10: Typical probability density functions used in economics[57].

[56] cf. Begg (2012). p. 22

The normal distribution is a bell shaped curve symmetrical about the mean μ, which is in this particular case equal to the mode. Hence μ has the highest frequency and each value with the same distance to the centre line has of course the same frequency[58]. This distribution has a significant importance for stock returns like in the modern portfolio theory or in real option models like Black & Scholes which assumes that returns are normal distributed. Still it has its limitation because of its symmetry. Gains are treated the same way as the risks of losses. Investors in the real world don't treat an upside and downside movement the same way as it is proposed by the normal distribution[59]. Moreover, with bell shaped curves it is also not possible to capture the current trend on the market. Most analysts prefer fat tailed distribution like lognormal or Cauchy because they allow higher frequencies for extreme values[60].

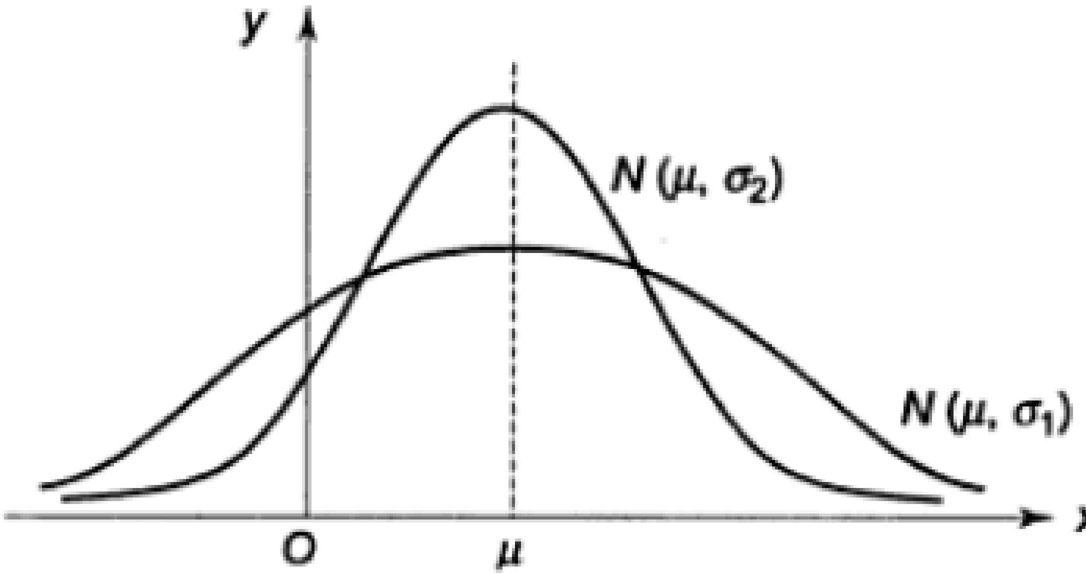


Fig. 2-11: The normal (or Gaussian) probability curve. The wider the distribution is, the larger the standard deviation and the higher the quantities of risk[61].

In case of the lognormal distribution the natural logarithm of a random variable is distributed along a bell curve. The advantage of this function is that it always leads to positive values and is therefore often used in the calculation of stock prices.

Basic RO models like Black & Scholes use a lognormal distribution for the underlying asset price. If you split up the stock price S into (infinitesimal) short intervals, the random behavior can be described as the previous stock price times the price change or return. Rather than using discrete compound rates, real options are continuously discounted and are therefore converted to a continuously compounded return Z which equals the natural logarithm of price changes over a period T (16).

$$(16)\ Z = \ln\left(\frac{S(T)}{S(0)}\right)$$

[57] cf. Begg (2012), p. 28
[58] cf. Veerarajan (2003), p. 216 f.
[59] URL: http://www.bankinvestmentconsultant.com/bic_issues/2010_2/rethinking-modern-portfolio-theory-2665530-1.html?zkPrintable=1&nopagination=1 (04.05.2013)
[60] URL: http://www.ehow.com/info_7982047_normal-probability-distribution-stocks.html, (04.05.2013)
[61] source: Veerarajan (2003), p. 218

According to the stated equation (16) the price change is lognormally distributed if the return Z follows a bell shaped curve[62]. Basically the only restrictions that need to be met to assure that the return Z is normally distributed are related to the so called efficient market hypothesis (=EMH).

E. F. Fama states that “a market in which prices always fully reflect available information is called efficient”[63]. Information goes hand in hand with risk, so in general terms the expected return is a function of its risk. If all information is fully utilized in determining the return the market is called efficient[64]. In other words the reaction of an investor when new relevant information is available is random and follows a normal distribution. Regardless if the investor’s decision was right or wrong, the stocks on the market are always perfectly priced according to the information. The chance that the price of a stock increases or decreases are random but equal. So no investor is able to consistently find under- or overvalued stocks because the deviations of the market price are ironed out immediately. According to the EMH no investment strategy is better than a coin toss.

On the contrary one can argue that if EMH allows for random events, absolute market efficiency is impossible. Furthermore, the various methods for valuing stocks are so different, that no one can ascertain the price on an efficient market. Besides not all investors possess the same information and they gain a wide range of different returns which cannot be explained with this theory. In addition it would not be possible to beat the market. Warren Buffett, CEO and chairman of Berkshire Hathaway is a famous example. His company significantly outperformed the average market over long periods of time and the returns are well above the traditional benchmark[65]. Therefore many authors believe that it is nearly unimaginable that the market will achieve perfect efficiency anytime soon[66,67].

Even if this theory is heavily controversial and often debated it is definitely suitable for a particular market and an average investor[68]. EMH is classified in three groups – the weak, the semi strong and the strong form. In its weak form the future prices cannot be predicted using historical information. On the other hand in the strong form all relevant information, public or private, is determining the price. Hence, the future stock price change is not dependent on the previous change and is distributed identically[69]. This leads to the random walk hypothesis (RWH) which is a consequence of the EMH. According to this principle the stock price changes randomly and is consequently unpredictable.

Other assumptions that needs to be met are related to the two already mentioned frequently used statistical measures, the mean μ (14) and variance σ^2 (15). If the stock price over time is divided into equally sized intervals h, then the mean and the variance must be independent of the interval length. In other words the expected continuously compounded return and the variance of this continuously compounded return can decrease or increase if h changes.

62 URL: http://waqqasfarooq.com/waqqasfarooq/index.php?option=com_content&view=article&id=63&Itemid=72 (12.05.2013)

63 Fama (1970), p. 383

64 cf. Fama (1970), p. 384 ff.

65 URL: http://www.warrenbuffett.com/ever-wondered-how-warren-buffett-beats-the-market/ (18.05.2013)

66 URL: http://www.forbes.com/sites/investopedia/2011/01/12/efficient-market-hypothesis-is-the-stock-market-efficient/2/ (19.05.2013)

67 URL: http://www.ft.com/cms/s/0/efc0e92e-8121-11de-92e7-00144feabdc0.html (20.05.2013)

68 URL: http://pages.stern.nyu.edu/~adamodar/pdfiles/eqnotes/mkteff.pdf (21.05.2013)

69 URL: http://waqqasfarooq.com/waqqasfarooq/index.php?option=com_content&view=article&id=63&Itemid=72 (12.05.2013)

Even if the distribution of the underlying stock price is not known, the central limit theorem says that if the number of random samples from the unknown distribution is large and identically distributed the distribution of the samples will tend to be normal. A sample size of approximately 50 is in most cases sufficient.

As already mentioned the rate of return follows a normal distribution and consequently for a time t the underlying asset price is in simplified terms a lognormal distribution (16).

$$(16)\ y = k * e^{r*t}$$

Generally it can be said that the more risks involved in a project, the larger the expected return, but the wider the distribution[70].

2.1.11 Risk and Uncertainty

According to the definition risk is the possibility of a deviation from a company's objective. It is the combination of the probability of occurrence and the impact on the organization. Risks are not entirely negative. The greater the risk, the higher the chances are. In other words risk is the variance of an expected outcome. Traditional evaluation methods add a risk premium on top of the discount rate to account for the degree of risk involved in the project. Therefore projects with high risks, even if they have great chances (e.g. follow up projects or a steep learning curve) will have a lower present value. Risks are typical symmetrical which means that on the one hand with a certain probability you lose money but on the other hand you also have a chance to gain money. For example a perfectly symmetrical risk could be that a stock goes down by 5 dollars with a probability of 50%. But with the same chance it also can go up by 5 dollars.

Uncertainties on the other hand are depending on unknown circumstances and a not quantifiable probability of occurrence. Moreover uncertainties increase with time because of an increasing number of possible outcomes over the life of a project. This is often presented as the cone of uncertainty.

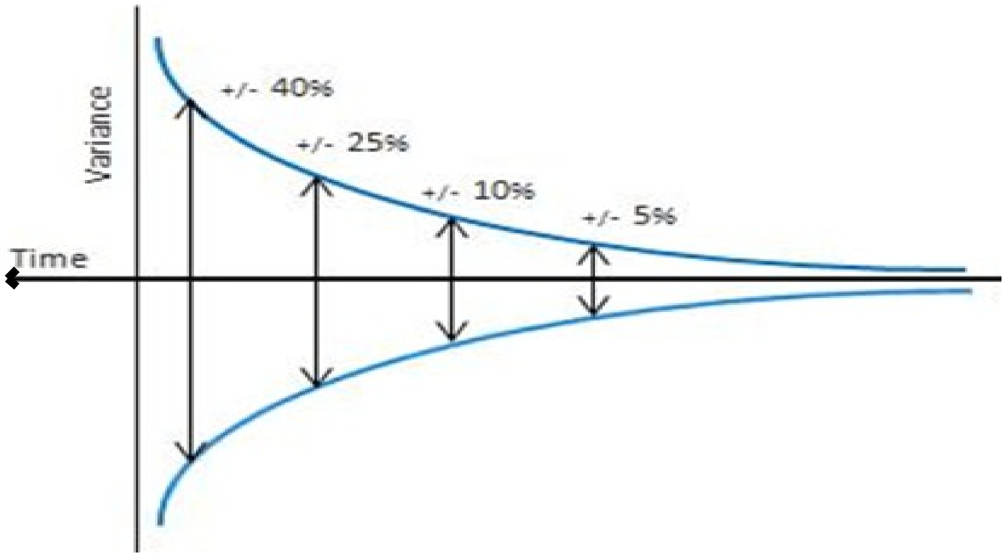


Fig. 2-12: The cone of uncertainty describes the increasing uncertainty with time[71].

[70] URL: http://www.ehow.com/info_7987619_distribution-stock-market-returns.html (05.05.2013)

[71] URL: http://www.modernanalyst.com/DesktopModules/DnnForge%20-%20NewsArticles/Print.aspx?tabid=128&tabmoduleid=889&articleId=2324&moduleId=630&PortalID=0 (23.02.2014)

The big advantage of real options is that they do not only focus on the downside, the rewards are also incorporated.

2.2 Origin of Real Options in the Mining Industry

Real options use the mathematics of financial options on real assets. One of the pioneers to utilize this concept was Brennan et al. 1985 in the mining industry. In his article he describes a case closely related to the oil and gas business.

Consider a gold mine which will produce a certain amount of raw materials at known costs. The costs itself can be discounted at the riskless interest rate, whereas the earnings can be evaluated using the futures market price under today's expectations. So in contrast to the discounted cash flow analysis in which the cash flows are calculated by adding up sales and costs over a specific period of time that will then be discounted to the present day, this new method further imbeds the current market and a riskless interest rate. The theory clearly defines boundaries and variables, is flexible and can therefore better reflect the real situation. With equation (17) the futures price is related to current spot price and accordingly to the market. As defined by Brennan et al. (1985) this equation must hold because even if the future price of gold is less, the owner will sell his gold today and enter into a futures contract to repurchase the sold gold.

$$(17)\ F_t = S_o * (1 + R)^t$$

In other words the price of a futures contract is determined by the spot price of the underlying asset value S_o, adjusted for the period of time t and the riskless interest rate R. Because one of the limitations is an arbitrage free market the riskless interest rate is allowed be used in equation (17).

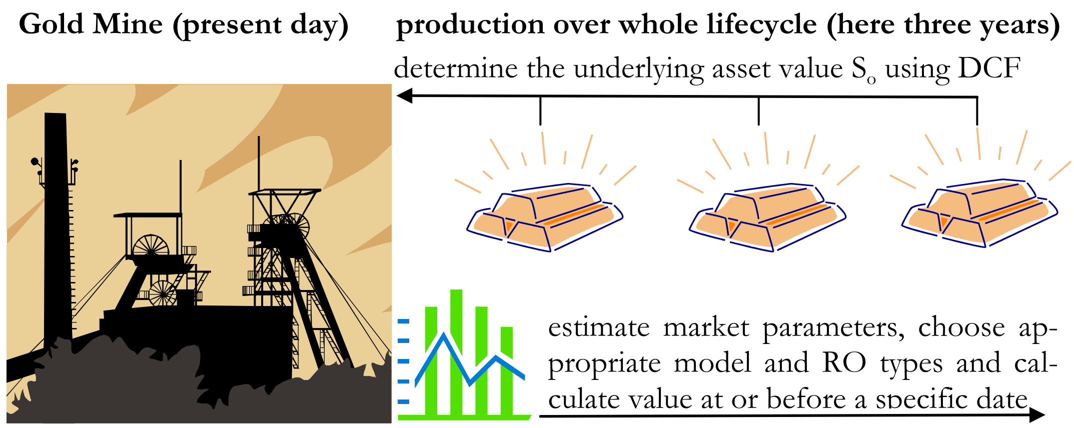


Fig. 2-13: Real options value of a gold mine.

As a first step the underlying asset value needs to be determined using the DCF method. Sharp declines or increases are difficult to account for in partial differential equations like Black & Scholes. If such "jumps" are expected the lattice models are much more applicable and easily allow appropriate corrections[72]. The second step incorporates the estimation of the current market parameters such as riskless interest rate or market volatility. Then the

[72] cf. Kodukula (2006), p. 86

equations according to the chosen model will be solved and the real options value be calculated.

Not everyone can make a profit of selling gold today because the noble metal needs to be held by someone and therefore the futures price is limited. Furthermore, it can also not exceed the value in equation (17) because otherwise one can make a fortune of purchasing gold today and at the same time selling future contracts. If the market is at equilibrium the present value can therefore be calculated by valuing the future output at the current spot price without discounting. Brennan et al. (1985) were the first to link the value of a project in this case a mine to market values. The spot price was the defined parameter to decide whether to abandon, temporarily close or delay the operations[73].

2.3 Real Options in the Oil and Gas Industry

Decision makers in the oil and gas business face a quite similar challenge every day. After acquiring a license for a specific area the company has the right but not the obligation (not considering commitments) to extract hydrocarbons from the underground. By using the market price of oil and gas including the restriction that the market needs to be at equilibrium and efficient, the future value of discrete events can be calculated.

Even if the mathematics behind real options was already defined by Stewart Myers in 1977 the applications of real option valuation to oil and gas investments are sparse. The reasons for this are most likely the various assumptions that are not spelled out and the different often contradictory approaches that are available.

Especially in the oil and gas business the investments are because of the volatile price and the various contract types, joint ventures, licenses and the inaccurately addressing uncertainties rather complex. Managers need to understand the strategic value behind uncertain assets that have a much larger range of possible outcomes. A proactive management can alter the course of a project and steer from possible losses to gains. Consequently it is important that a method is established that also accounts for value creation which corresponds to the upside aspect of uncertainty.

Typically for E&P are large amounts of capital and high uncertainties, hence the chances are vast and projects can generate enormous values. Flexibility is the key parameter that is mainly affected by outside uncertainty and intrinsic complexity of the investment under consideration. The different types of options are defined and limited by the project itself. Common flexibilities in an E&P project are wait-to-invest, termination, temporary start/stop as well as operational related issues. The classic stage gate process that is often used in the E&P industry can further categorize real options and act as an overview of possible opportunities over the life cycle of a field (Fig. 2-14). Each stage is followed by a decision making opportunity that will be supported by the real options approach[74].

[73] cf. Brennan et al. (1985), p. 41 f.

[74] cf. Jafarizadeh (2009), p. 3 ff.

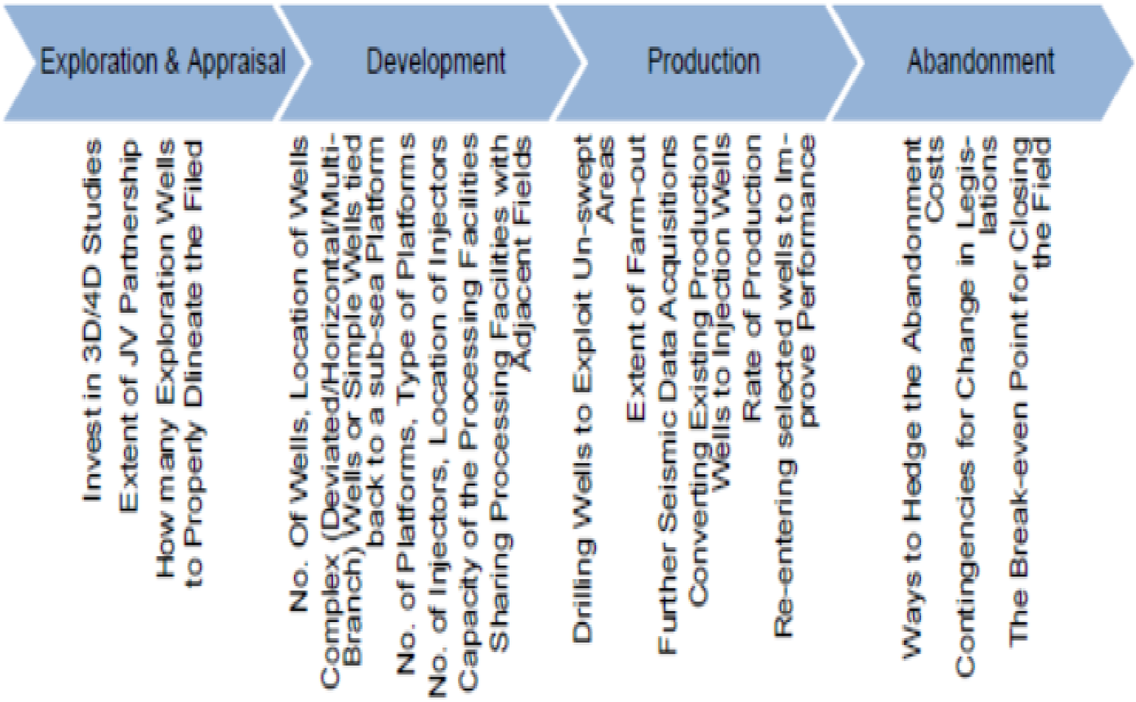


Fig. 2-14: Common real options during the life cycla of a petroleum field[75].

2.4 Decision Making Process in E&P

Decision makers are often confronted with sub-optimal evaluation of projects in the past and are seeking for new methods to fully account for imbedded uncertainties. A typical reason for wrong evaluations is a culture of optimism instead of critical estimation of incorporated risks and uncertainties. Case studies show that it is important to fully understand the main uncertainties and consider as well as plan for many possible futures. It is often easier to focus on issues that can be analyzed rather than on the overall performance.

The current business climate with demanding investors and a variety of market risks are key drivers for a new approach. Generally decisions in the E&P world can be divided into three main categories – policy, strategic and operational related decisions[76] (Fig. 2-15).

[75] cf. Jafarizadeh (2009), p. 4
[76] cf. Begg (2012), p. 9 ff.

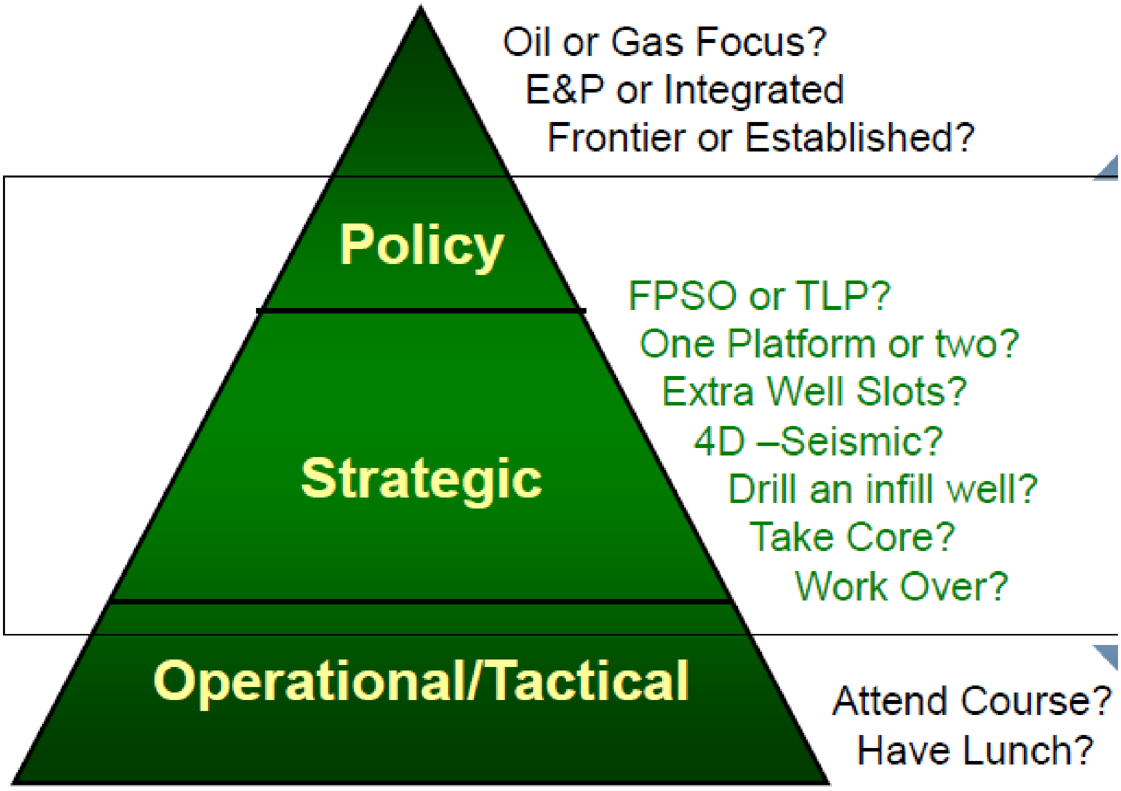


Fig. 2-15: Three main types of decisions with examples[77].

A decision itself should be defined as a conscious, irrevocable allocation of resources to achieve a desired objective. It is important to know the objective ("what you want"), the possible choices ("what you can do"), the information about uncertain and not controllable events ("what you know"), the impacts ("what you will/wont get") and the decision criteria ("how to choose"). Good decisions are not based solely on the actual outcome but on the process and the consideration of all potential events. A proper model can replace confusion by clear insight and offer a procedure to integrate the major aspects in a systematic way. A careful application of a decision analysis based on a model will lead to better decisions and better outcomes[78].

Typical for E&P are large investments with unknown returns. Moreover, results are based on models that can be wrong and data to reduce uncertainty is usually expensive. Especially geology and geophysics offer a wide variety of alternatives and the real options approach can help to capture all possible outcomes. Some uncertainties that might affect a decision are for example:

- formation water salinities in the calculation of hydrocarbon saturations
- chosen Archie parameters
- selected matrix densities and clay properties
- relative permeabilities and properties of oil and gas
- contacts like the oil water contact and the free water level
- seismic velocity models
- flow properties

[77] cf. Begg (2012), p. 23.
[78] cf. Bratvold et al. (2009), p. 21 ff.

- drilling issues e.g. due to overpressured zones
- required facilities capacity
- net reservoir/net pay definition
- fiscal terms
- oil and gas price
- efficiency of late life opportunities
- or simply bad weather …[79]

To accurately represent the situation real options need to be carefully implemented. For every single stage in the life cycle of a field the appropriate real option value need to be calculated and compared to other key figures. In most projects companies may have more than one alternative so more complex options like compound or rainbow options should be chosen to adequately represent the given situation. Nevertheless even simple options can give further insight and estimate the incorporated value of flexibility to justify advanced types of options.

[79] cf. Begg (2012), p. 26 ff.

3 Real Option Models and Types

Real options evolved from financial options. The classical methods use the same setup of financial options to evaluate real assets. Even if some limitations and the source of the uncertainties might be different, this approach provides a first insight. Especially in case of high uncertainties and therefore high flexibility the added value from the real options valuation technique can be substantial.

The next chapter will give a short overview of financial options and the commonly used terms and models. Financial options built a solid framework for the real options and define the significant input parameters. Because financial options are based on market information and don't account for private risks new methods like the market asset disclaimer and the so called integrated approach were developed to better mimic the real world.

Nevertheless except some rare cases the mathematics for financial options remains and provides a good understanding for the limitations of a model.

If the appropriate model for a given situation is selected and if necessary the mathematics adopted it is important to define the flexibility of the management. According to the possible outcomes the types of real options can be selected. For example a very common option is the option to wait till some market or private uncertainties will clear. This option can create additional value that can then be evaluated using the defined model. This value of flexibility cannot be adequately estimated using the discounted cash flow analysis because only a go or no-go decision can be implemented. However according to the market asset disclaimer approach the net present value can provide valuable information that should be incorporated in the real options technique.

Both steps the selection of a proper model and the types of options are crucial and responsible for an accurate evaluation of the economic value of a project.

3.1 Financial Options: the Origin of Real Options

3.1.1 Put and Call Options

Financial options are basically a security on the markets. It is a simple binding contract two parties, the option seller and the option buyer sign. Upon the agreed terms and conditions the buyer has the right, but not the obligation, to buy or sell an underlying asset at a specific price on or before a certain date.

There are basically two fundamental financial options. An option to buy, also called call option and an option to sell which is called put option. Because an option is a right but not an obligation the price at which the option is exercised (= exercise or strike price) is, not considering the options premium, always a positive value. This premium is a fixed value defined by the options seller which needs to be paid by the options buyer in order to acquire the predefined right. Moreover this premium reflects the risk for the seller involved in such a deal. If the option is of an American style it can be exercised on or any time before the maturity or expiration date. A European style option on the other hand can only be exercised at the expiration date, i.e. at a single point in time[80].

The value of a call option is calculated by determining the maximum of zero and the difference between the underlying asset value S and the exercise price X at the time when the asset is bought (18).

$$(18)\ V_{call} = \text{Max}\ [0, S - X]$$

On the contrary the value of a put option is the maximum of zero and the difference between the exercise price and the underlying asset value at the time when the asset is sold (19).

$$(19)\ V_{put} = \text{Max}\ [0, X - S]$$

If in case of a call option S-X is below zero, the underlying asset value is less than the strike price and consequently will not be exercised. In this situation the options is considered to be "out of the money". In the same manner a put option is called to be "out of the money" if the exercise price is less than the asset value. The option will definitely not be exercised and the value is therefore zero. In contrast to options that are "in the money" where S-X and X-S is greater than zero, respectively. The net payoff primarily depends on the options premium. When the asset value is exactly equal to the strike price, the option is considered to be "at the money"[81].

The following examples should illustrate the defined terms and create a further understanding. The presented cases are possible decisions in the oil and gas industry. In the first example the investment costs of a new horizontal drilling technology will be evaluated. In the second case a petroleum company examines buying a new process plant to increase their throughput in Libya.

In the first example consider a research institute that has invented a new technology for precise directional drilling for long lateral sections. Because the institute is not sure whether the future test will be successful, management decides to sell the patent to an American shale gas company. This company already has enough experience to make use of such a patent even if the test is not a success. The research institute declares that they are willing

[80] URL: http://www.investopedia.com/articles/optioninvestor/07/options_beat_market.asp (05.08.2013)
[81] cf. Kodukula (2006), p. 3 ff.

to pay a predefined premium of € 20 million in order to be able to sell the intellectual property for € 80 million within the next five years.

There are basically three possible situations. Because the tests failed the value of the technology stays below the € 80 million and the institute exercises the option and sells their patent. This is an American style put option which is in this case "in the money" (Fig. 3-1). In a second situation the expected value of the technology after five years is € 100 million, the net payoff is zero and the option is called to be "at the money". In the third scenario the tests were a full success and the value of the technology is far beyond the € 80 million. Hence, the put option is "out of the money" and will not be exercised.

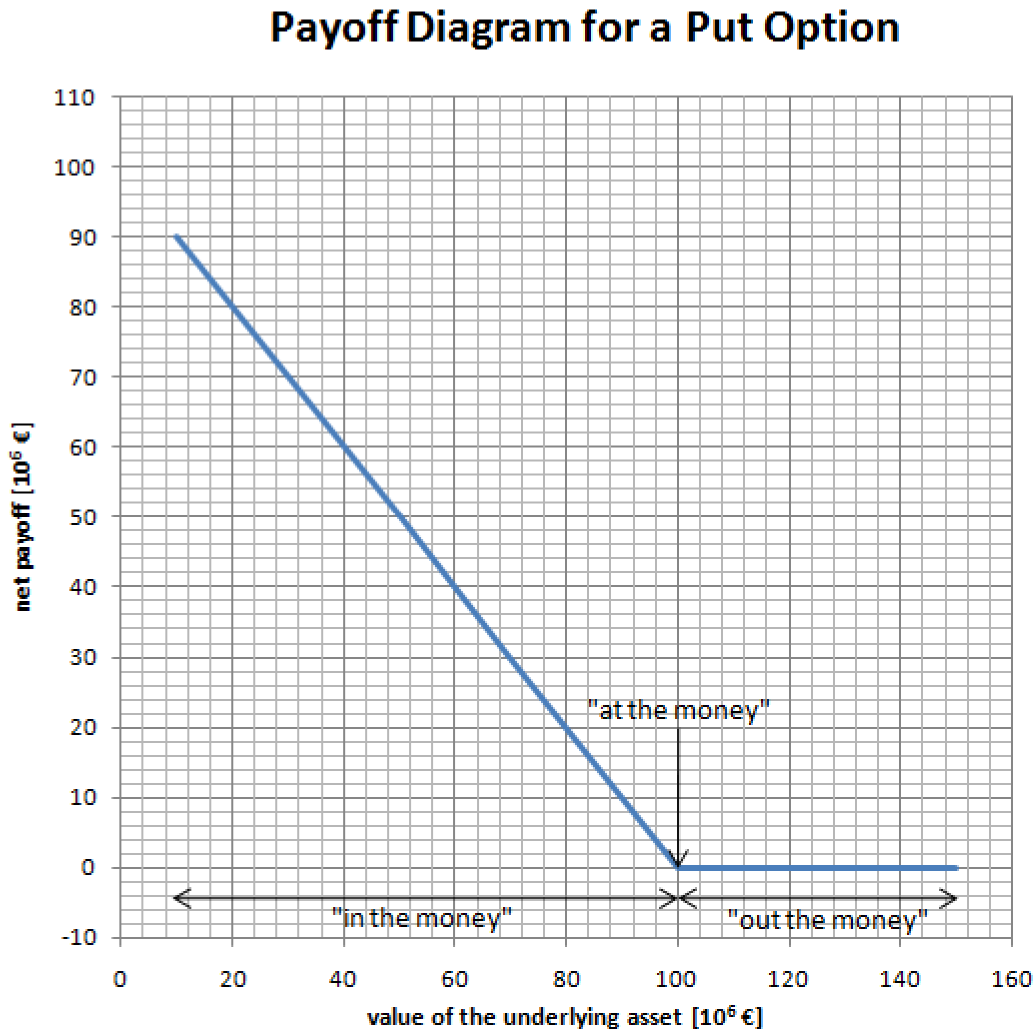


Fig. 3-1: Payoff diagram for an American put option.

FuelAM, a petroleum company, would like to acquire a € 300 million offered gas processing plant operated by GasP in Libya. Because of political and market uncertainties the company doesn`t want to commit to the full investment at the present time. Management decides to create an option to expand and invest € 30 million in the plant today with the possibility to buy the whole asset at any time within the next three years.

Again there are three possible situations presented in Fig. 3-2. A civil war erupts and because of political instabilities FuelAM decides not to exercise the option. In this case the American style call option is "out of the money". In another scenario the country flourished and gas prices are on the rise again. The expected asset value is above the sum of the premium and the investment. Consequently the call option is “in the money” and FuelAM decides to exercise the option and buy the gas processing plant. If the sum of the premium and the investment (= € 330 million) is equal to the asset value then the option is called to be “at the money”.

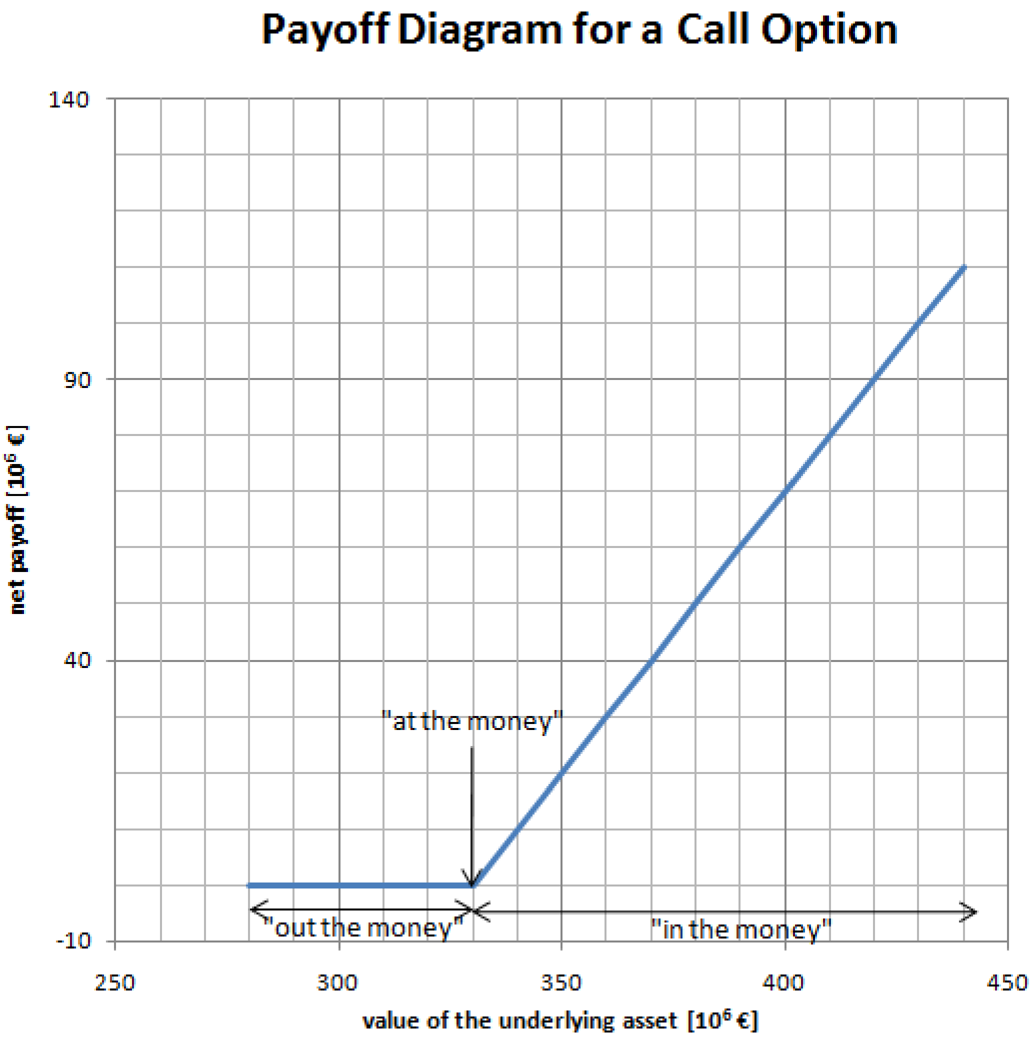


Fig. 3-2: Payoff diagram for an American call option.

3.1.2 Information, Time and Uncertainty

Like in the discounted cash flow analysis time has a major impact on the economic value but not only because of the time value concept but also because uncertainties may be reduced. So a simple “waiting” has in the options way of thinking a positive impact on the project. Uncertainties are closely linked to the value of flexibility. If there are no uncertainties there is no flexibility and the NPV is an adequate tool to measure the economic value[82].

In general information increases with time and hence some uncertainties will reduce or even vanish while the asset value and the project payoff will change significantly either in a

[82] cf. Kodukula (2006), p. 126 f.

positive or negative way. In the simple models where prices follow a geometric Brownian motion this is often expressed in a binomial tree. An up-movement will increase the economic value because the uncertainty evolved to a chance whereas a down movement will indicate that an uncertainty turned out to be a loss and accordingly reduced the economic value of the project[83].

But in order to capture all possible paths one must create a stochastic process that can calculate a composite outcome with the assumption that decision makers are only interested in maximizing the value. So if the asset falls below a certain value the project should be immediately abundant. On the other hand if the market develops favorably and technical risks can be reduced to a minimum the project value increases. However, management needs some flexibility to respond to fluctuations in the market, otherwise the company is not able to maximize asset values and opportunity costs incur. This can be represented with the cone of uncertainty in Fig. 3-3. The discounted cash flow analysis can only capture the lower part of the cone. If there is no risk and uncertainty involved in a project the pay-off is certain and the cone collapses to a straight line. The angle between the "high" and "low" line illustrates the degree of uncertainty. Even if the cone is usually symmetrical it doesn`t mean that every outcome has the same probability. The overall distribution of all paths in real options can be calculated via the number of paths that lead to that result and the probabilities along the way[84].

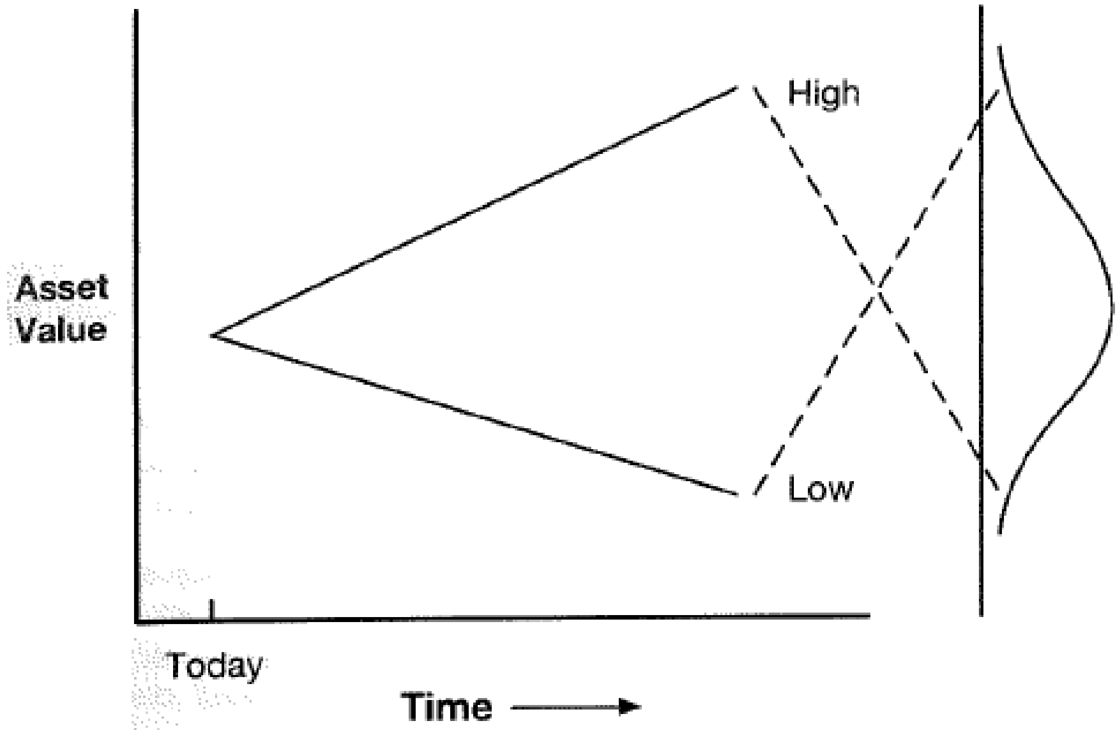


Fig. 3-3: The cone of uncertainty and the distribution of all possible outcomes[85].

[83] cf. Kodukula (2006), p. 57 ff.
[84] cf. Kodukula (2006), p. 56 f.
[85] cf. Kodukula (2006), p. 57

3.1.3 Traditional Discounted Cash Flow vs. Real Options Approach

If the discounted cash flow methods are directly compared to the modern real options approach there are several distinct differences concerning flexibility, the sources of uncertainty as well as their application and requirements.

First of all the DCF is an all or nothing strategy - a go or no-go decision. Based on a static single discount rate management needs to decide whether to invest in a project now or let the opportunity pass by. With real options you can calculate the value if you defer the project for some time or you alter the course of a project. Managerial and time flexibility as well as uncertainty are key factors for RO. Especially in long term projects the strategic value needs to be considered and projects with high uncertainty but great opportunities will adequately be estimated. A simple discounted cash flow analysis could according to the agreed discount rate under- or overestimate the project`s economics. In most cases managers will react risk averse and select a too high premium. The higher the risk adjusted discount rate the greater the losses over time. But risky ventures also offer great opportunities and therefore chances to increase the asset value over the life of a project. These options are not considered in the traditional DCF. On the other hand if a certain goal needs to be achieved it could even be the way around. To meet the requirements managers will be overall optimistic and add only a very marginal risk premium. The bottom line is that in both cases the discounted cash flow analysis will not be representative[86].

In general the risk premium in the DCF added to the discount rate accounts for the involved risk whereas in RO payoff itself is adjusted for risk and consequently expressed as the probability distribution of the payoff. Since the risk adjusted premium is as well static over the life time of a project, only one situation can be incorporated in the DCF. Besides the investment costs are in the same way discounted as the payoff[87].

Additionally Pritsch & Weber 2003 mention the increased perception of the surroundings as a result of the process of determining a real options value. Because many possible situations are weighed against each other the resultant distribution gives a detailed insight[88].

According to Meise 1997 the main differences of financial and real options are exclusivity, compoundess and tradability. In the financial market only the option holder has the exclusive right to exercise the option whereas with real assets another company can exercise exactly the same option. Compound options as one class of options that depend on other options are in the financial market referred to as exotic options[89] (e.g. chooser option) and are very seldom used. Moreover, most of the real assets are not tradable, have a longer time till maturity and net convenience yield are compared to dividends that vary over time[90].

[86] cf. Kodukula (2006), p. 56 f.
[87] cf. Kodukula (2006), p. 58
[88] cf. Pritsch (2003), p. 149 transl.
[89] cf. Clewlow (1997), p. 50
[90] cf. Meise (1997), p. 55 transl.

Table 3-1: Comparison of the traditional discounted cash flow analysis and the real options approach[91].

	Real Options	Discounted Cash Flow
time	period of time	point in time
flexibility	over a period of time	at a point in time
strategy	dynamic	static
cash flows	asymmetrical	symmetrical
uncertainty	changing	given

Another important difference is the environment for which the different models were built for. This can typically indicate the limitations of the model. Financial options are needless to say built for the financial markets whereas real options were developed for real assets in a risky environment including market and private uncertainties. Real options can now either follow the principle of the financial options or they differ from the mathematics of financial options in such a way that input parameters don`t solely depend on the market anymore.

In principal the value of a share or a tradable product can according to the market hypothesis randomly go up and down. As long as the returns are normally distributed (see also central limit theorem) an options value can be calculated with the classical approach (e.g. any analytical solution of the PDE like Black and Scholes). If the asset of a project is fully tradable on the market and the price follows a geometric Brownian motion this set of equations can also be used to determine the value of a project. The advantage is that the theory is already in place and if the requirements are met, a value can easily and fast be calculated. Whereas the underlying asset of financial options are mainly stocks and bonds traded on the market, the assets of real options may include real estates, intellectual properties, projects, ... which are usually not traded.

Without adopting the model real options will often only be able estimate the economics in case regulations, technical risks or the time needed to realize an action is not considered.

Some of those disadvantages can nowadays be overcome (e.g. combination of decision tree analyses and real options) in the more sophisticated methods. Still it will never replace experience and intuition but it can help to create a key value for managerial flexibility and uncertainty.

Real options are of particular interest if a project has a high market and in case of the integrated approach also a high private uncertainty. With the necessary flexibility the high upside potential can be capitalized. In case a project already is deep in the money (e.g. high NPV) or is not economic at all, the ROA will not add any significant information. So real options should only be calculated if projects are close to be economic but offer high potential. Particularly if two projects have a very similar NPV, real options can make a huge difference.

[91] cf. Hilpisch (2006), p. 59 transl.

3.1.4 Active and Passive Management

It might be of interest for a company to defer the project and proactively invest some money to clear the market uncertainty. These relatively small investments can alter the economics of a project in a positive way. This procedure is known as active management. On the contrary a company can also decide to wait and let time resolve uncertainty. In this case management believes in the efficient market hypothesis and minimizes investing fees and costs. This is on the other hand called passive management[92].

3.1.5 Terminology of Financial and Real Options

Financial options establish the basic mathematical theory and the equations so that real options can be used to evaluate projects by maximizing the output and minimizing the opportunity costs. Since real options evolved from financial options both more or less share the same terminology.

The exercise price or strike price is the defined price of an option a buyer has to pay the seller for the underlying asset. Moreover the stock price in real options called the underlying asset value is simple the estimated value of the current asset. For example in the market asset disclaimer the underlying asset value can be the net present value of the asset. The expiration time is the period of time for which the option is valid. After the expiration date the option cannot be exercised anymore. In case of a European option the security can only be exercised on a specific date in contrast to American option that can be exercised on any day within the given time frame. The volatility is the variation in the price of an asset and can reflect market, private (e.g. technical, geological uncertainties) or both uncertainties. The risk free interest rate is defined by long term government bonds because the risk a government is going bankrupt is zero – or very close to zero. So there is no risk involved and hence the rate of the bonds is called the risk free rate. The option premium or the option price is a price at which the security can be bought on the market. It reflects the involved risk for the seller. Finally the dividends or in real options the convenience yield is the benefit of physically holding the asset. For example the typical convenience yield for crude oil is the highest in the winter, whilst those for agricultural commodities are the highest in the initial stage of the harvest period[93]. A summarize of the terminology of financial and real options can be found in table 3-2 on the next page.

[92] URL: http://www.investopedia.com/university/quality-mutual-fund/chp6-fund-mgmt/ (03.11.2013)

[93] cf. Kodukula (2006), p. 4 f.

Table 3-2: Terminology of financial and real options and abbreviations[94].

Terminology	Financial Options/Real Options	Abbreviations
exercise price strike price	the predefined price in the options contract to buy or sell a financial/nonfinancial asset (= investment costs)	X
stock price	the value of the underlying asset	S
expiration time	the time till the option expires	T
volatility	variations of the price of an asset, the higher the volatility the greater the value of an option, is a measurement of the involved risk	σ
risk free interest rate	the interest rate of long term government bonds that are considered to be without any risk	r
option premium option price	the price that has to be paid to acquire the option	P
dividends	payment to the shareholders/convenience yield	D

[94] cf. Kodukula (2006), p. 3 f.

3.2 Real Option Models

Compared to the traditional evaluation methods like NPV or the decision tree analysis (= DTA), the real options approach is far more complex. Moreover, for the practical use it is important to know the limitations. Since Stewart Myer introduced the term real options in 1997 many different models were developed to solve a variety of flexibilities and sources of uncertainty. One of the most crucial steps is not only the selection of the right model but also the appropriate determination of the input parameters like volatility or the underlying asset value.

Because real options are rather an enhancement of the DCF than a replacement, usually the first step is the calculation of the underlying asset value. In the market asset disclaimer model this is done by discounting the determined cash flows of a project using a risk adjusted discount rate (= NPV). The next step incorporates the estimation of the investment costs, the degree and sources of uncertainty and the corresponding flexibilities over a defined period of time. The selection of the correct model depends first and foremost on the desired simplicity, the available input data and the validity of the method itself.

In general the available real option models can be divided into two groups. The first group are analytical approximations where the solution of the partial differential equations (= PDE) describing the change of the value of an option over time is given by one equation (e.g. Black and Scholes for European options). A partial differential equation basically is a function of independent variables and their partial derivatives. These equations are typically solved by defining boundary conditions like

$$(20)\ V_{call} = \max(0, S - X)$$

and $$(21)\ V_{put} = \max(0, X - S)$$

in case of Black and Scholes[95].

These easy to handle analytical solutions of the PDE are often used to give a first estimate of the result. They help to gain a global picture without the need of great mathematical skills.

Probably the most famous equation to calculate the value of European call and put options was invented by Fischer Black, Robert Merton and Myron Scholes (= BS) in 1973. Even if most real options are of American type this closed form solution provides an estimate of the economic value in no time[96].

Analytical solutions for American type options are far more complex and till today no precise equation for long-term options exist. Barone-Adesi and Whaley (1987) were the first to provide a fast quadratic analytical approximation[97]. The drawback is the inaccuracy, especially for long maturity options[98]. Recently Cheng and Zhang (2012) invented a new analytical pricing method as an extension of the formula of Cheng et al. (2010) for American options with dividend yields using the homotopy analysis method[99]. With this equation it is possible to calculate accurate option values for up to two years of time to maturity using

95 cf. Kodukula (2006), p. 66 f.
96 cf. Hilpisch (2006), p. 94 transl.
97 cf. Barone-Adesi et al. (1987), p. 302
98 cf. Ju et al. (1999), p. 32 f.
99 cf. Cheng et al. (2010), p. 1148 f.

the put call symmetry between the two basic types of options. Although it is only applicable if the dividend yield is smaller than the risk free interest rate[100].

Because of their restricted application and for detailed insight numerical methods are prioritized by decision makers. These numerical techniques can be again grouped into three classes (table 3-3).

The lattice methods like the binomial method described by Cox, Ross and Rubinstein[101] as an example that calculates the option value in discrete time intervals.

Another technique is the usage of Monte Carlo simulations (=MCS) to determine the options value. With MCS it is possible to simulate different paths of the project payoff. So it is not necessary to define the PDE. The average value of every calculated path is the desired options value[102]. The quasi Monte Carlo method on the other hand systematically chooses points in the defined space rather than determining the value of randomly selected paths[103]. Especially if several sources of uncertainty are incorporated other models than MCS face difficulties and are not practical anymore[104].

The third quite common numerical method is based on finite differences. First the PDE are approximated by a finite number of points. In a second step the derivative at each point will be transformed into a set of finite difference equations. There are again two different methods to solve the system. The implicit method solves the PDE indirectly by solving a system of simultaneous linear equations. On the other hand the PDE can also be solved directly through small intervals backward in time. This is known as the explicit method[105].

But the available methods cannot only be classified according the solution of the PDE but also according to the sources of uncertainty and how they incorporate market and private uncertainties.

Besides many different approaches two are commonly found in the literature. Especially in a high risk environment with different sources of uncertainty many authors recommend the so called integrated approach.

Next to the already described classical methods like Black and Scholes the market asset disclaimer is also very popular. This approach simply uses a Monte Carlo Simulation of the traditional net present value without flexibility to define all input parameters. Important is the by Copeland and Antikarov (2001)[106] described link to the current market. The discount rate is typically derived from the weighted average cost of capital and this is with the beta value directly connected over a replicating portfolio with the market. In case the geometric Brownian motion can describe the price fluctuations the market asset disclaimer can accurately estimate the economic value of the project including flexibility by using the discounted cash flow analysis as a basis and the real option techniques as an add on[107].

The second frequently used method is that of the integrated approach. Here the sources of uncertainty are separated from each other and as a first step evaluated differently. After the combined volatility is estimated the underlying asset value as well as the options value is calculated.

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[100] cf. Cheng et al. (2012), p. 160 ff.
[101] cf. Cox et al. (1979), p. 229 ff.
[102] cf. Kodukula (2006), p. 68
[103] cf. Acworth et al. (1996), p. 3 f.
[104] URL: http://marcoagd.usuarios.rdc.puc-rio.br/monte-carlo.html#quasi-MC, (02.07.2013)
[105] URL: http://marcoagd.usuarios.rdc.puc-rio.br/katia-num.html, (04.07.2013)
[106] cf. Copeland et al. (2001), p. 90 f.
[107] cf. Jafarizadeh et al. (2009), p. 3 f.

Table 3-3: Overview of available real options analysis solution methods[108].

Techniques	Specific Methods
analytical techniques (closed form solutions)	European options: e.g. Black & Scholes 1973[109] American options: e.g. Barone Adesi & Whaley 1987[110]
numerical techniques	lattices: binomial, trinomial, quadrinomial, multinomial, e.g. Cox et al. 1979[111] Monte Carlo simulations: normal and quasi, e.g. Gamba 2003[112] finite difference methods: implicit and explicit, e.g. Cortazar 2000[113]

Every single method needs to be implemented with care and the models limitations should be known and considered. Nevertheless the proper calculation of real options can be very time consuming and if the project at hand only has very limited flexibility the difference between this rather new method and the traditional net present value will diminish.

3.2.1 Analytical Techniques: Black and Scholes

The expected return μ of a project is simply the sum of the payout rate δ and the expected rate of capital gain[114].

$$(22)\ \mu = \delta + \text{expected rate of capital gain}$$

In other words the expected return is the average net outcome that can be calculated using the rate of return in a certain time period and the dividends as a fraction of the net income that will be paid to the stockholders.

$$(23)\ \frac{\Delta S}{S} = (\mu - \delta)\Delta t$$

If there is no risk involved than the expected return equals the risk free interest rate. On the other hand if the project is risky the change of the value can be described by a normally distributed random process with a mean of zero and a standard deviation of one[115].

$$(24)\ \frac{\Delta S}{S} = (\mu - \delta)\Delta t + \sigma * \Delta e_t$$

108 cf. Haug (2006), p. 23 ff.
109 cf. Black et al. (1973), p. 637
110 cf. Barone-Adesi et al. (1987), p. 301
111 cf. Cox et al. (1979), p. 229
112 URL: http://papers.ssrn.com/sol3/papers.cfm?abstract_id=302613, (05.07.2013)
113 URL: http://papers.ssrn.com/sol3/papers.cfm?abstract_id=251653, (05.07.2013)
114 URL: http://web.mit.edu/rpindyck/www/courses.htm, (11.07.2013)
115 URL: http://web.mit.edu/rpindyck/www/courses.htm, (12.07.2013)

The second term is defined by a Wiener process and represents the noise around the general trend described by the first term[116].

A Wiener process is called a generalized Brownian motion if it has the form

$$(25)\ dx = a(x,t) * dt + b(x,t) * dz$$

The drift is defined by a(x,t) and the scattering is specified by the variance parameter b(x,t)[117].

In short a Brownian motion is nothing else than a set of random variables B_t with $0 \leq t \leq T$ which are normally distributed for each t and are independent of each other. The result dx is normally distributed with the expected value $\varepsilon(dx)=a(x,t)*dt$ and the variance $\upsilon(dx)=b^2(x,t)*dt$. Because

$$(26)\ dz = e_t * \sqrt{dt}$$

is the increment of the motion and if

$$(27)\ a(x,t) = \mu * x$$

$$(28)\ b(x,t) = \sigma * x$$

the above equation can be rewritten as

$$(29)\ dx = \mu * x * dt + \sigma * x * dz$$

$$(30)\ dS = \mu * S * dt + \sigma * S * dz$$

$$(31)\ \frac{dS}{S} = \mu * dt + \sigma * dz$$

whereas dS/S is normally and therefore dS lognormally distributed[118]. This is because the change of z over n intervals of length Δt in the Wiener process can be according to the central limit theorem (= CLT) represented by a normal distribution with a mean of zero and a standard deviation of one. The above equation is also known as the geometric Brownian motion or the Ito process[119].

A simulation of the geometric Brownian motion is illustrated in Fig 3-4. The corresponding Excel spreadsheet with the stated equations can be provided by the author of the master thesis at hand.

With the given parameters μ and σ the general trend of the upper and lower boundary of the cone of uncertainty was calculated. The figure shows one example of a possible path of a geometric Brownian motion with a general drift illustrated by the red line.

[116] cf. Hull (1993), p. 190 ff.

[117] cf. Dixit et al. (1994), p. 63.

[118] cf. Dixit et al. (1994), p. 70 f.

[119] cf. Chance (1994), p. 41 f.

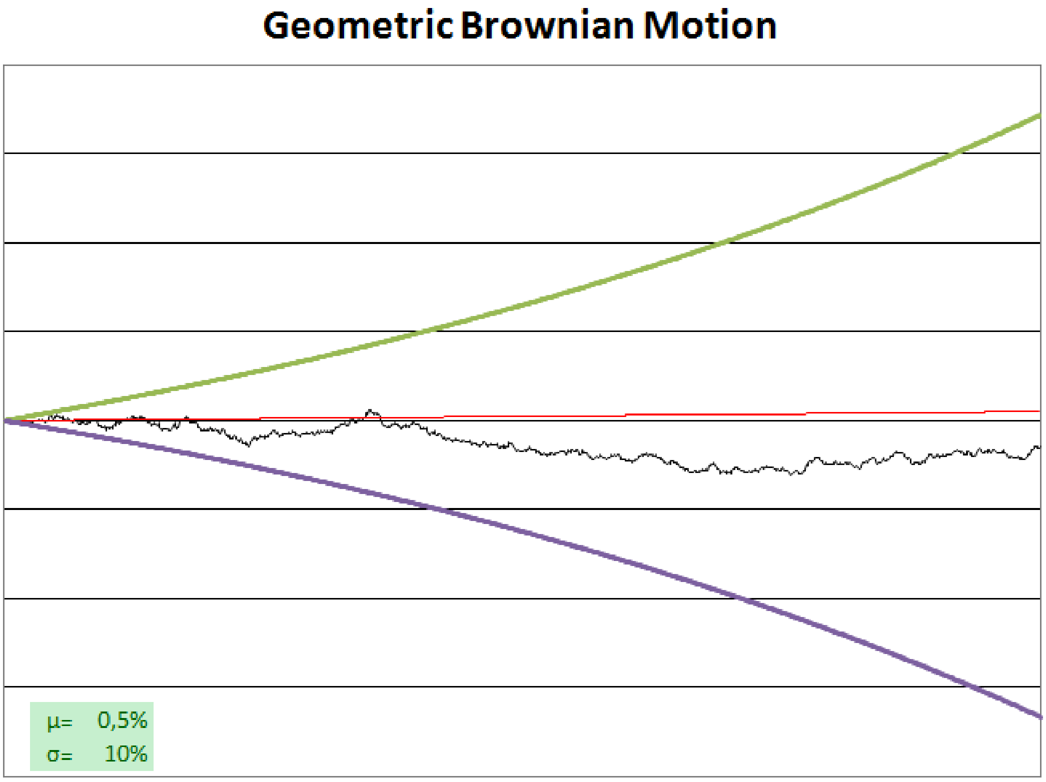

Fig. 3-4: The geometric Brownian motion within the general trend[120].

Equation (31) is the fundamental partial differential equation of the closed form solution by Black and Scholes. So if the stock price follows a geometric Brownian motion like the one in Fig 3-4 the rate of return is normally distributed and with BS an analytical solution can be found. In order to demonstrate the mathematical procedure it is necessary to define Ito`s Lemma first. This lemma is used to find the differential of a time dependent function of a stochastic process. It can be compared to the chain rule for determining the derivative of a function[121].

By using a Taylor series in x and t the derivatives of S(x, t) are determined.

$$(32)\ dS = \frac{\delta S}{\delta x} * dx + \frac{\delta S}{\delta t} * dt + \frac{1}{2}\frac{\delta^2 S}{\delta S^2} * dx^2 + \frac{1}{6}\frac{\delta^3 S}{\delta x^3} * dx^3 + \ \ldots$$

Furthermore, dx and dx^2 is calculated and substituted in the equation above.

$$(33)\ dS = \frac{\delta S}{\delta x} * dx + \frac{\delta S}{\delta t} * dt + \frac{1}{2}\frac{\delta^2 S}{\delta S^2} * dx^2 + \frac{1}{6}\frac{\delta^3 S}{\delta x^3} * dx^3 +$$

$$(34)\ dx = \ a(x,t) * dt + b(x,t) * dz$$

with $dz = \ \sqrt{dt}$ according to the Wiener process

$$(35)\ dx^2 \ = \ a(x,t)^2 * dt^2 + 2 * a(x,t) * dt * b(x,t) * \sqrt{dt} + b(x,t)^2 * (\sqrt{dt})^2$$

[120] URL: http://www.wiwi.uni-muenster.de/vwt/Veranstaltungen/EnergieseminarSS06, (12.07.2013)

[121] cf. Dixit et al. (1994), p. 79 f.

Taking into account that $dt^{1,5}$ and dt^2 are approaching zero much faster than dt, these terms can be neglected. Thus resulting in:

$$(36)\ dx^2 = b(x,t)^2 * dt$$

For dx^3 the power of dt will be greater than 1 and therefore goes to zero faster than dt. Thus the differential dS is

$$(37)\ dS = \frac{\delta S}{\delta x} * dx + \frac{\delta S}{\delta t} * dt + \frac{1}{2}\frac{\delta^2 S}{\delta x^2} * dx^2$$

and by substituting for dx

$$(38)\ dS = \left[\frac{\delta S}{\delta t} + a(x,t) * \frac{\delta S}{\delta x} + \frac{1}{2} b(x,t)^2 * \frac{\delta^2 S}{\delta x^2}\right] * dt + b(x,t) * \frac{\delta S}{\delta x} * dz$$

The above equation is called Ito`s Lemma and defines the correct expression for differentials which depend on the Brownian motion[122].

Since in the geometric Brownian motion a function for sake of simplicity call it G follows a lognormal distribution according to

$$(39)\ G(x) = \ln S$$

the derivatives are

$$(40)\ \frac{\delta G}{\delta t} = 0$$

$$(41)\ \frac{\delta G}{\delta x} = \frac{1}{x}$$

$$(42)\ \frac{\delta^2 G}{\delta x^2} = -\frac{1}{x^2}$$

Referring to Ito`s Lemma

$$(43)\ dG = \frac{1}{x} * dx + 0 * dt + \frac{1}{2} * (-\frac{1}{x^2}) * dx^2$$

with $(29)\ dx = \mu * x * dt + \sigma * x * dz$

and $(36)\ dx^2 = \sigma^2 * dt$

$$(45)\ dG = \mu * dt + \sigma * dz - \frac{1}{2} * \sigma^2 * dt$$

$$(46)\ dG = (\mu - \frac{1}{2} * \sigma^2) * dt + \sigma * dz$$

Now if G(x) = ln S then

$$(47)\ dlnS = lnS_{t+dt} - lnS_t = \ln\left(\frac{S_{t+dt}}{S_t}\right) = \left(\mu - \frac{1}{2} * \sigma^2\right) * dt + \sigma * dz$$

which proofs that the rate of return is normally distributed with

$$(48)\ \ln\left(\frac{S_{t+dt}}{S_t}\right) \sim N[\left(\mu - \frac{1}{2} * \sigma^2\right) * dt, \sigma * dz]$$

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122 URL: http://www.math.tamu.edu/~stecher/, (15.07.2013)

and

$$(49)\ \ln(S_{t+dt}) \sim N[\ln S_t + \left(\mu - \frac{1}{2} * \sigma^2\right) * dt, \sigma^2 * dt]$$

$$(50)\ S_{t+dt} = S_t * e^Y \text{ with } Y = \left(\mu - \frac{1}{2} * \sigma^2\right) * dt + \sigma * e_t * \sqrt{dt}$$

To actually calculate the call option value Black and Scholes used the following boundary conditions

$$(51)\ C = \max(0, S - X)$$

and $(52)\ P = \max(0, X - S)$.

In case of a call option this means that if the future price is greater than the price stated in the option then value is S-X, otherwise its zero.

But to actually derive the equations by Black and Scholes it is necessary to define the term replicating portfolio and risk neutral as well as their interdependence first.

The Risk Neutral Evaluation

Both techniques the replicating portfolio and the risk neutral evaluation need to deliver identical results if based on the same assumptions but the mathematics is slightly different. The principles are based on a binomial lattice.

This binomial process states that the underlying asset can only have two distinct states – up or down. So S_0 as the current prize of the underlying asset can in the following period either be S_{up} or S_{down}. The value is determined using the multipliers u and d, respectively. These multipliers are dependent on the volatility of the share price and the time increment[123].

The main difference between the two techniques is that the risk neutral approach derives the result from the intrinsic value of the later two nodes of the binomial tree. The price of a share today is the present value of the expected future values. The risk neutral technique calculates the needed multipliers p and q for given future values to get the present price through what is known as backward induction (Fig. 3-5) [124].

[123] cf. Dixit et al. (1994), p. 114 f.

[124] cf. Kodukula (2006), p. 72 f.

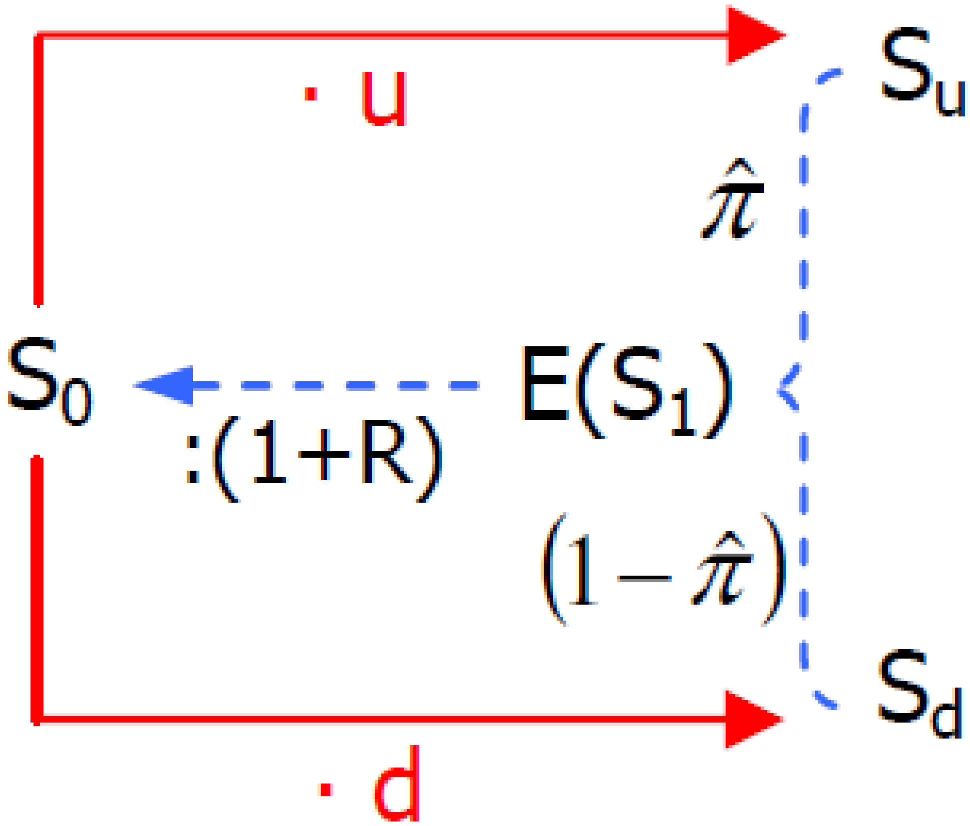

Fig. 3-5: The principle of the risk neutral evaluation[125].

These multipliers, even if they are often called risk neutral probabilities or Martingale factors are weighting coefficients rather than probabilities. They are risk neutral because the expected value is discounted using the risk free rate without any risk premium.

$$(53)\ S = \frac{p * S_u + (1 - p) * S_d}{1 + r}$$

$$(1)\ S_u = S * u$$

$$(1)\ S_d = S * d$$

$$(54)\ S = \frac{p * S * u + (1 - p) * S * d}{1 + r}$$

$$(55)\ p = \frac{(1 + r) - d}{u - d}$$

The Replicating Portfolio

On today's market there are a huge variety of assets with different returns and risks traded. If it is possible to create a portfolio of existing traded assets so that it replicates the return and risk structure the price of the asset will correlate with the market value of that portfolio. The uncertainty involved in such a portfolio can typically be described by a geometric Brownian motion or in other words an Ito process[126].

[125] URL: http://www.tiberian.ch/files/opt.pdf, (16.07.2013)

[126] cf. Dixit et al. (1995), p. 114 f.

This so called replicating portfolio is a hedging strategy that comprises of a certain number of underlying stocks and risk free bonds.

Let's assume a riskless investment where the stock is balanced with the option value and define the upper and lower boundaries of a call and put option. At time zero you buy a stock at a value of S, write a European call C and borrow the present value of the strike price X of the option. At the maturity there are two possibilities. If S<X the call option will be worthless, you sell the stock and pay the loan back. On the other hand if S>X the option holder exercises the call and you have to pay the difference. The investment is riskless and also in this case the total will even out (table 3-4).

Table 3-4: The cash flow of a riskless investment in case of a European call option.

	Today	Time T, S<X	Time T, S≥X
buy a stock	-S	S	S
borrow present value of X	$X*e^{-r*T}$	-X	-X
write call option	C	0	-(S-X)
total cash flow	$-S+X*e^{-r*T}+C$	S-X≤0	0

Because it is assumed that the market is arbitrage free the cash flow at time zero must be positive and so the call option value is

$$(56) -S + X * e^{-r*T} + C \geq 0$$

This equation can be rearranged to give the boundaries of a call option.

$$(57) C \geq \max(S - X * e^{-r*T}, 0)$$

On the contrary if you short sell the stock, write the put option and invest the present value of X, there are again two possibilities at expiration. If S>X the option is worthless because the market offers a higher price than the one stated in the option. So take the money from the investment and use it to buy the asset. Otherwise you take the money from the investment and exercise the option and buy the stock (table 3-5).

Table 3-5: The cash flow of a riskless investment in case of a European put option.

	Today	Time T, S<X	Time T, S≥X
sell a stock	S	-S	-S
lend present value of X	$-X*e^{-r*T}$	X	X
write put option	P	-(X-S)	0
total cash flow	$-S+X*e^{-r*T}+C$	0	X-S≤0

This defines the boundaries of the put option value.

$$(58) P \geq \max(X * e^{-r*T} - S, 0)$$

If both cases are now combined so that you short the asset, sell the put, buy the call and invest the present value of the strike price X the result is known as the put-call parity (= PCP).

In other words, the value of European put and call options for the same asset with the same strike price X and expiration date T are related according to the put-call parity (= PCP).

$$(59)\ C - P = S - \text{present value}\ (X)$$

If inequality exists the market is not arbitrage-free and the equations by BS are not adequate[127].

American put options generally have a greater value than the European counterpart except for call options. In case of a call option the value of an American call option equals the value of a European call option if both have the same strike price and expiration date. The difference between American and European options is the ability to exercise the option earlier which is not profitable. The reason is that at any time t<T the option has according to the equation

$$(60) C \geq \max(S - X * e^{-r*(T-t)}, 0)$$

for American options a lower value. Consequently an American call option will only be exercised at expiration T like a European option and not before.

The replicating portfolio is now a method to combine a riskless borrowing in case of a call option or a lending in case of a put option with a certain number of underlying stocks so that the portfolio has the same cash flow as the option. As shown in the example above a gain/loss from stocks is balanced by a loss/gain in option value. Hence the portfolio is riskless and if the market is arbitrage-free all cash flows can be discounted using the risk-free rate[128].

Call Option = borrowing + buying a specific number of the underlying stock
Put Option = lending + selling a specific number of the underlying stock

So the value of a riskless portfolio according to the up and down movement of a binomial process are

$$(61)\ P_u = m * S_u + B * (1 + r)$$

$$(62)\ P_d = m * S_d + B * (1 + r)$$

where the first term describes the number of shares of the underlying stock and the second term is the return of a riskless bond (Fig. 3-6).

[127] URL: http://www.math.tamu.edu/~stecher/, (15.07.2013)
[128] cf. Dixit et al. (1994), p. 77 f.

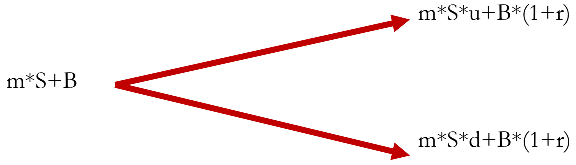

Fig. 3-6: The principle of the replicating portfolio.

Because the portfolio tracks the value of the option

$$(63)\ P_u = C_u$$

$$(64)\ P_d = C_d$$

in case of a call option. The system of equations can now be solved for m and B.

$$(65)\ m = \frac{C_u - C_d}{S_u - S_d}$$

$$(66)\ B = \frac{S_u * C_d - S_d * C_u}{(S_u - S_d) * (1 + r)}$$

$$(67)\ S_u = S * u$$

$$(68)\ S_d = S * d$$

$$(69)\ m = \frac{C_u - C_d}{S * (u - d)}$$

$$(70)\ B = \frac{u * C_d - d * C_u}{(u - d) * (1 + r)}$$

$$(71)\ C = m * s + B$$

$$(72)\ C = \frac{C_u - C_d}{(u - d)} + \frac{u * C_d - d * C_u}{(u - d) * (1 + r)}$$

If a continuous risk-free rate is used, the term (1+r) is simply replaced by e^{-r}.

Black and Scholes: the Final Equation

With the appropriate risk neutral probabilities or the parameters of the replicating portfolio the formula by Black and Scholes can be derived. In case of continuous discounting the current call option price is the discounted expected value.

$$(73)\ C = e^{-r*T} * E[\max(S_t - X, 0)]$$

S is according to the limitations by BS a random variable with a lognormal distribution with the following parameters.

$$(74) \ln(S_{t+dt}) \sim N[\ln S_t + \left(\mu - \frac{1}{2} * \sigma^2\right) * dt, \sigma^2 * dt]$$

After several transformations[129] this will yield the analytically solved BS equation for European call options. With the put-call parity it is now easy to also give an expression for the value of a put option.

$$(75)\ C = S * N(d_1) - X * e^{-r*T} * N(d_2)$$

$$(76)\ P = X * e^{-r*T} * \left(1 - N(d_2)\right) - S * (1 - N(d_1))$$

$$(77)\ d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r + \frac{\sigma^2}{2}\right) * T}{\sigma * \sqrt{T}}$$

$$(78)\ d_2 = d_1 - \sigma * \sqrt{T}$$

In general the process of Black and Scholes has four distinct characteristics:

- time independent stochastic changes define a completely random walk, so no predictions are possible
- continuous in time
- changes are normally distributed
- valid for constant μ and σ[130]

3.2.2 Numerical Techniques: The Binomial Lattice

The different lattices look like a decision tree analysis that starts with one value at the present time and evolves at a specific time increment to the expiration date.

Each node represents the underlying asset value at that time. If the optimal decisions at each node are combined in a role back algorithm the best possible economic value can be estimated. In such a case all risks turned out to be chances for the company and instead of a loss. By combined every single possible branch the distribution of the outcome can be calculated.

For investment projects with high uncertainties most authors prefer the discrete binomial model[131]. Even if it can only calculate values within a discrete time interval the model converges quite fast to the more accurate continues techniques so that the error lies within 1%[132].

The current underlying asset value is the starting point and at each time increment it can either go up or down according to the factors u and d. Both factors primarily depend on the volatility of the underlying asset and the time increment.

After the first time step the two nodes have the asset values S*u and S*d. The second time step results in three nodes in which the center node is the same for its upper predecessor`s downward movement and its lower predecessor upward movement. Such a lattice is called recombining[133].

In contrast a nonrecombining tree has two different center nodes as illustrated in Fig 3-8.

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[129] a detailed transformation is found in Hull (1993), p. 268-270.

[130] URL: http://www.investopedia.com/university/options-pricing/black-scholes-model.asp, (25.07.2013)

[131] cf. Zettl (2000), p. 1 f.

[132] cf. Burns (1992), p. 8

[133] cf. Kodukula (2006), p. 97 ff.

In general the calculation of a binomial lattice can be carried out using a six step process:

1. frame the application: identify the option type and incorporated flexibilities
2. identify input parameters: name and estimate the necessary input parameters especially the underlying asset value, strike price, option life, volatility, risk free interest rate and time increments
3. calculate needed option parameters including u and d as well as risk neutral probabilities
4. build the binomial lattice and calculate the corresponding asset value at each node
5. determine the option value at each node by backward induction
6. analyze the result and compare to other economic evaluations[134]

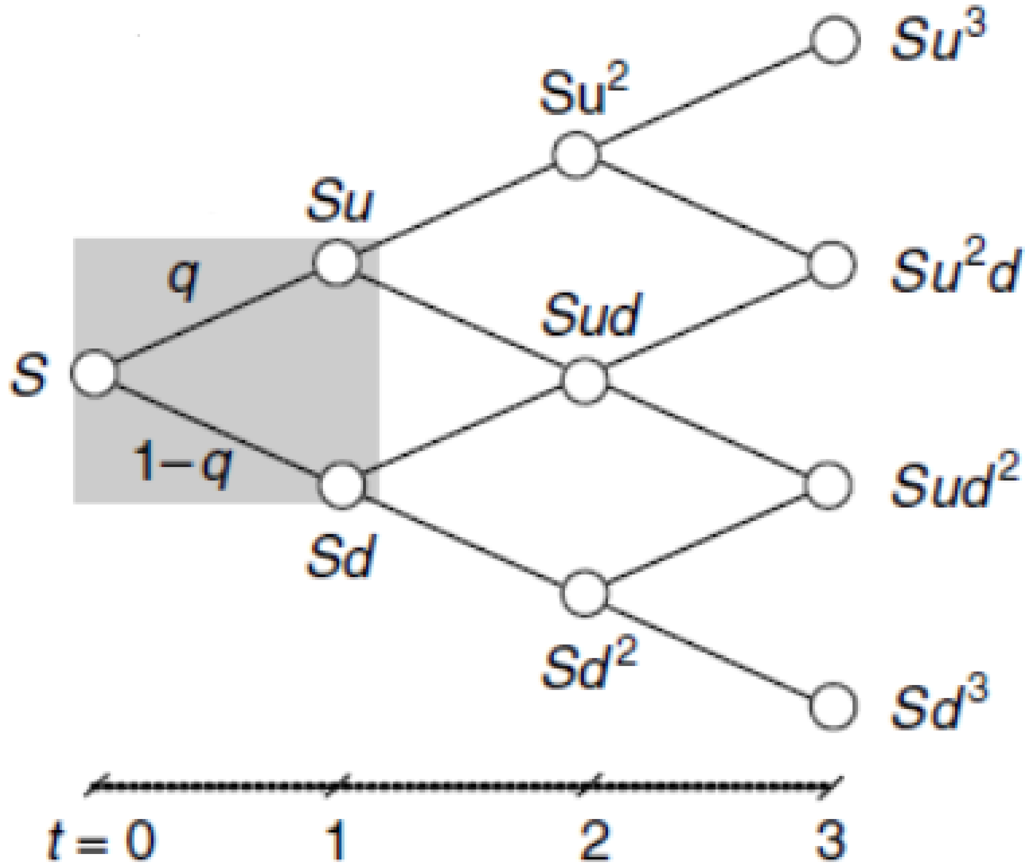

Fig. 3-7: A recombining binomial lattice[135].

[134] cf. Kodukula (2006), p. 97 f.
[135] cf. Brandao et al. (2005), p. 71

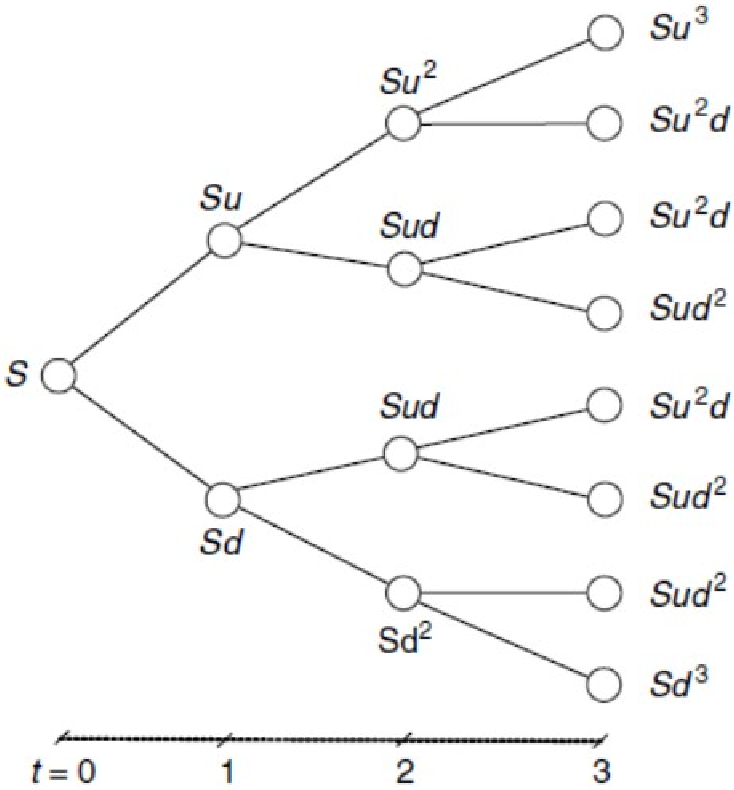

Fig. 3-8: A nonrecombining binomial lattice[136].

[136] cf. Brandao et al. (2005), p. 71

3.3 Comparison of Different Analytical and Numerical Evaluation Techniques

To compare the frequently used evaluation techniques and to determine the needed time increments the value of a call option with parameters specified in Table 3-6 was calculated with different analytical and numerical methods.

Table 3-6: Chosen parameters for the evaluation with different techniques.

Input		
S	100	€
X	120	€
T	5	a
σ	10	%
r	0,5	%

The net present value as the most important key figure of the discounted cash flow analysis is merely the sum of the discounted cash flows. Because the underlying asset value already represents the sum of the over the time generated earnings, the net present value can be calculated by the subtraction of S and the investment costs X. Because X is greater than S the NPV is negative. Nevertheless the project`s volatility is high and it can still turn out to be economic if the value of flexibility is considered. The only way to incorporate flexibility in a systematic way is the real options approach.

The formula by Black and Scholes which is only valid for European options is probably the easiest and fastest one to calculate an economic value. It can also be used as a first approximation and represents a lower limit. If the value calculated by Black and Scholes is already positive all other evaluation techniques will also definitely be positive. So before more complicated methods like Barone-Adesi & Whaley for American options or Bjerksund & Stensland are used this technique should give a good first approximation.

According to the given equations after Black and Scholes an Excel spreadsheet was constructed to determine the call and put option value without dividends. Additionally the put-call parity was implemented. As long as the price of the PCP and the prices calculated with Black and Scholes are equal no riskless arbitrage opportunities are possible. With this spreadsheet one can quickly estimate the value of real options. As already mentioned it will represent a lower limit because Black & Scholes is only valid for European options which do not offer the same high flexibility as American options. Caution is advised if the underlying asset value or project value is above a certain limit because then it can fall below the intrinsic value of the option (Fig. 3-9). This is because the project value of the remaining time to maturity is lower than the present value of the investment[137].

[137] cf. Hilpisch (2006), p. 100 f.

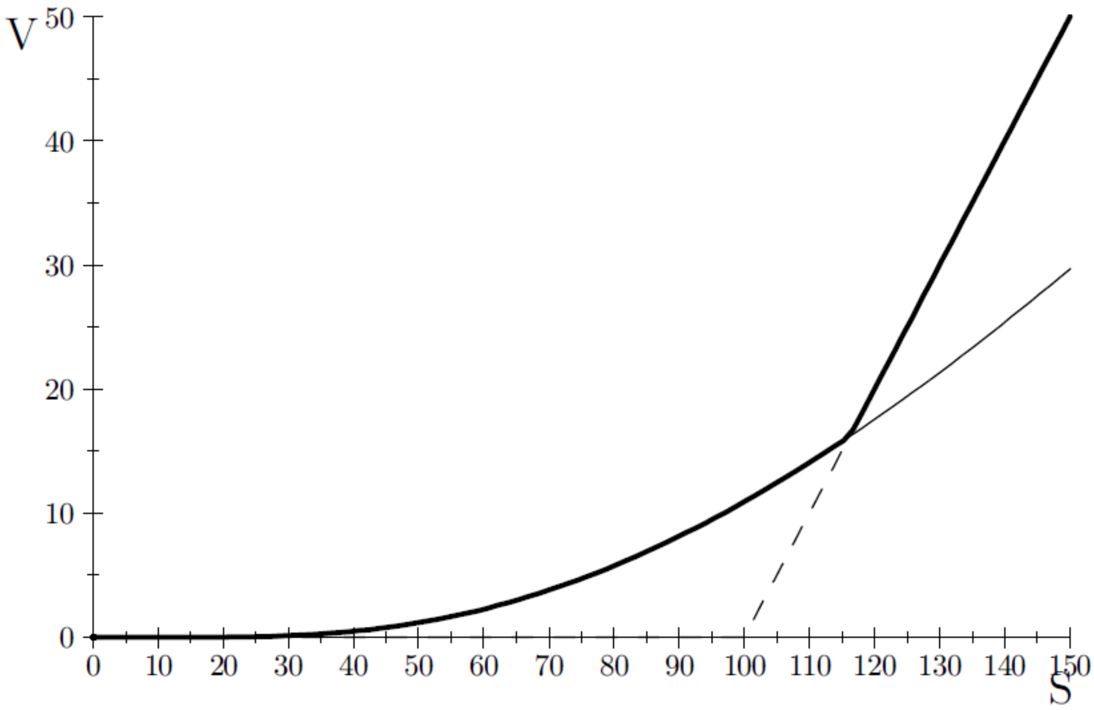

Fig. 3-9: The value of an option calculated with Black and Scholes can fall below the intrinsic value (dashed line). In this case it is recommended to use the intrinsic value instead (bold line)[138].

Therefore it is recommended to use Black and Scholes as a rule of thumb for the usual American type real options. By using the maximum value of Black and Scholes and the intrinsic value as the underlying asset value at time t minus the exercise price a wrong estimation even at high project values is prevented.

$$(79)\ V_{low} = \max[V_t^{BS}, S_t - X]$$

Basically the positive call options value indicates that even if the exercise price or the investment is above the project value the flexibility is high enough to give a positive RO value. Considering the net present value alone the project would today have a negative value of -20 € (S-X=NPV) and probably management would decide not to invest in the project. Because of not incorporated flexibility the discounted cash flow analyses will underestimate the actual value which will very likely lead to wrong decisions.

If the project is deep in the money or far out of the money the underlying asset value is the essential parameter and RO are close to DCF. The two asymptotes, namely the x-axis and the straight line corresponding to the intrinsic value in Fig. 3-10 demonstrate this behavior and also show the maximum value of flexibility. Only if a specific underlying asset value is reached real options slowly increase to positive values. Moreover real options can never have negative values because an option is a right and not an obligation to take an action on a nonfinancial asset. Hence, whenever the exercise of an option is not of advantage it will simply expire without any loss or profit, except the already paid option premium. Therefore the first asymptote is converging against the x-axis of the graph.

[138] cf. Hilpisch (2006), p. 102

The dashed line on the other hand represents the intrinsic value of the option (Fig. 3-9). This value can be easily calculated as the underlying asset value over time minus the investment. In other words the intrinsic value is the value that the option would have if it would be exercised today. It is a time independent value and defines the amount at which the exercising price is in the money[139].

The difference between the option price and the intrinsic value is called the time value or extrinsic value. It is directly related to the amount of time left until maturity. The greater this number is the higher the potential that an option will end up in the money. Hence, it is an indication about the flexibility of a project. The maximum is at an underlying asset value which is exactly the exercise price. If an option is at the money so that S – X ~ 0 the flexibility value reaches a maximum. A general financial rule states that an option losses one-third of its value during the first half of its life and the remaining two-thirds in the second period[140].

The second convergence is against the intrinsic value of an option. If an option is deep in the money, the ROA will not add any significant value.

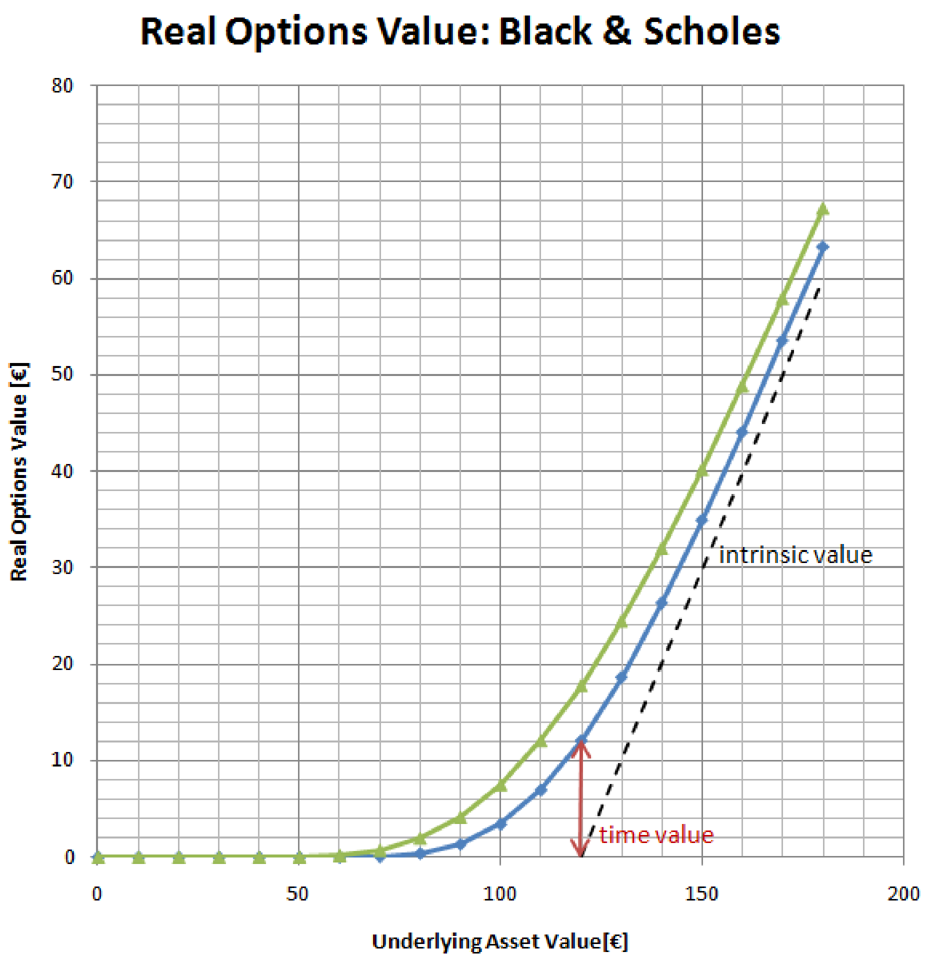

Fig. 3-10: The real option value as a function of the underlying asset value with 5 (blue line) and 10 years (green line) of expiration time.

[139] URL: http://www.investopedia.com/articles/optioninvestor/07/options_beat_market.asp (10.07.2013)
[140] URL: http://www.investopedia.com/articles/optioninvestor/02/021302.asp (10.07.2013)

In this simple example the difference between the NPV and RO calculated according to Black and Scholes are nearly 20% of the total investment costs and instead of rejecting the project there are good chances of getting approval by the management.

Several analytical and numerical evaluation techniques like Black and Scholes, the binomial lattice approach, the Monte Carlo based simulation, the explicit finite element method, Barone-Adesi and Whaley as well as Bjerksund and Stensland where compared with each other. The input parameters were the same as in the example above. Every calculated step of the simulation is represented by a corresponding dot on the diagram (Fig. 3-11).

Black and Scholes are slightly showing a lower value for the option than the other techniques. Because a European option has a reduced flexibility the lower value is expected. The wide range of dots indicates that the Monte Carlo simulation needed the highest number of steps to actually get to the final value.

Additionally for Black and Scholes a spider diagram was constructed. It clearly shows that the underlying asset value as well as the strike price have for the given example by far the most significant impact on the outcome (Fig. 3-12).

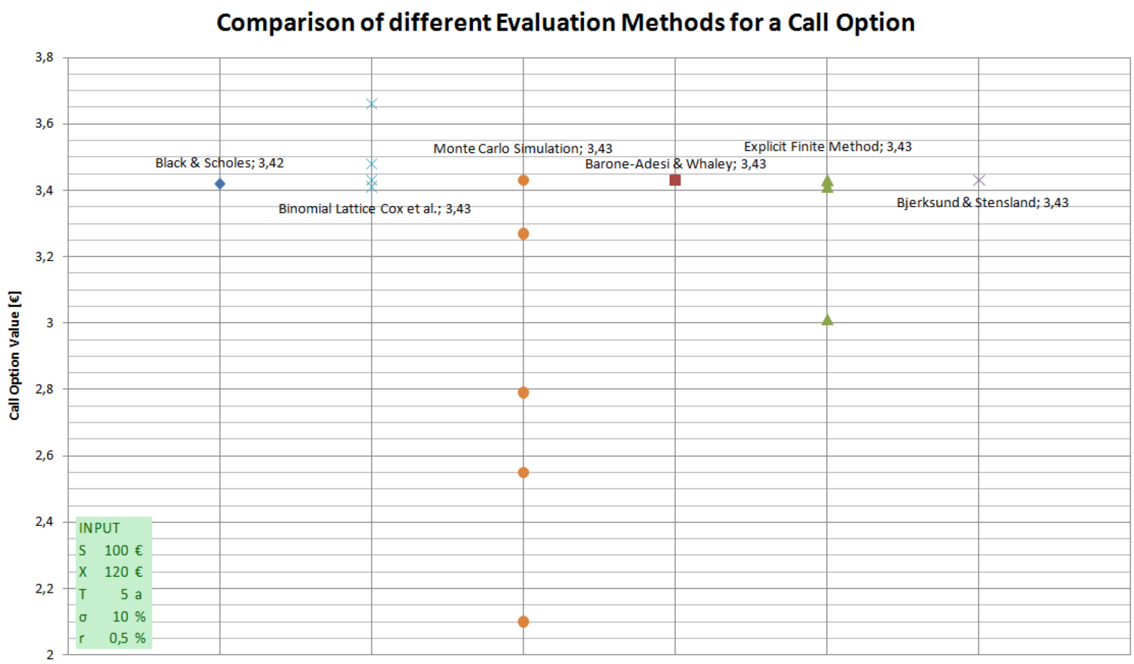

Fig. 3-11: Comparison of different analytical and numerical evaluation techniques.

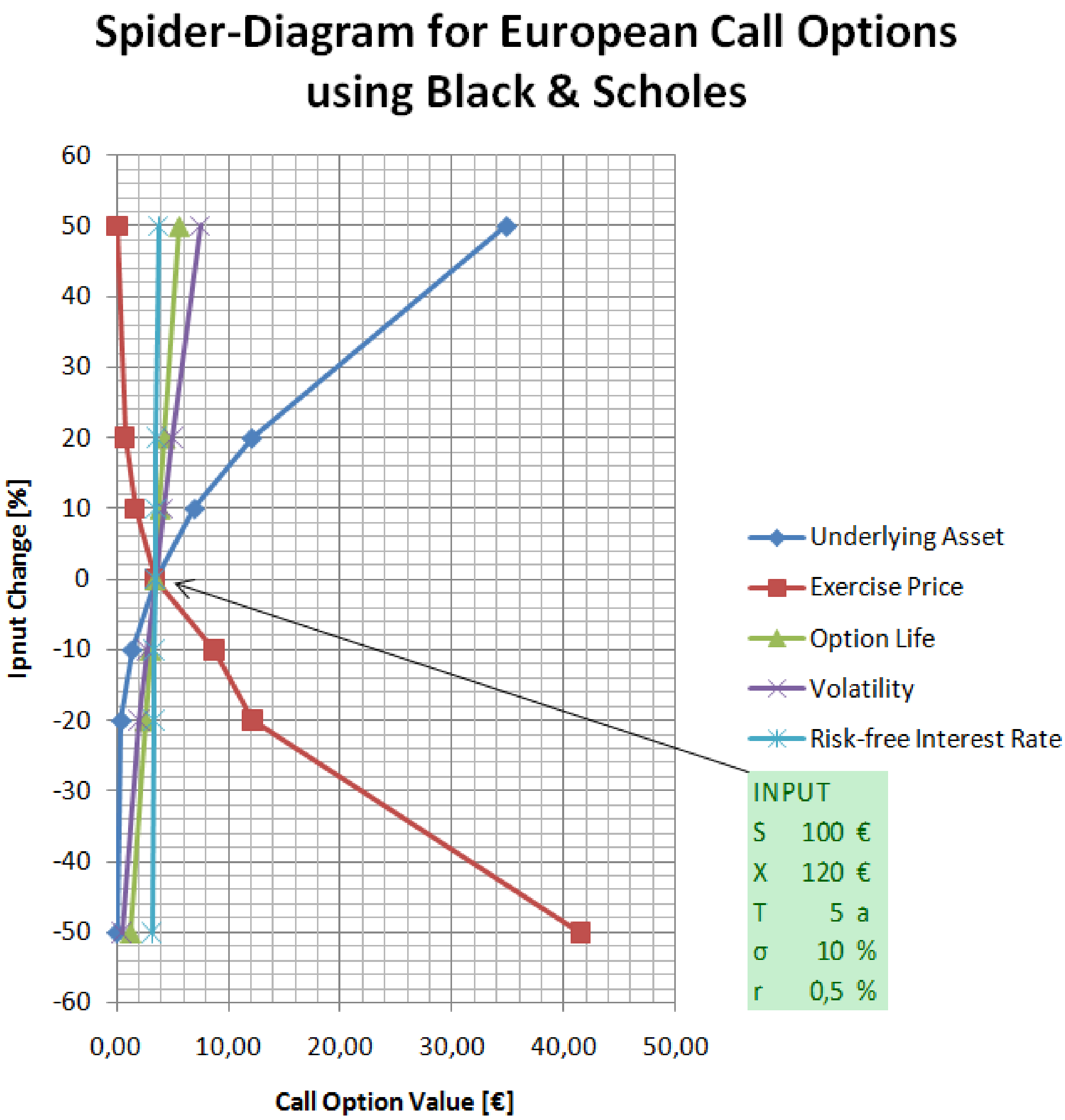

Fig. 3-12: Sensitivity analysis of Black and Scholes.

3.4 Types of Options

For decision makers it is important to know the different types of real options and their consequences. Each type has its own calculation scheme.

In general real options can be classified into two basic groups. The first group consists of simple options concerning more basic business actions. Options to expand to other business segments or options to defer a project to a later point in time are examples of this type of options. On the other hand there are so called compound options because their value depends on the value of another option instead of the underlying asset value. They are very common in multiphase projects for example in the petroleum exploration and production business (= E&P). Rainbow options - very special real options - are characterized by multiple sources of uncertainty and can be either simple or compound[141].

Table 3-7: Overview of the most common types of options[142,143].

Group	Option Types
simple	option to expand, option to switch, option to contract, option to abandon, option to choose, option to alter, option to extend, option to diversify, option to extend, scale-up option, scale-down option, switch-up option, scope-up option, scope-down option, option to wait, option to defer, staging option, option to shorten, …
compound	parallel compound option, sequential compound option, option to learn, rainbow option

[141] cf. Kodukula (2006), p. 61 f.
[142] cf. Hilpisch (2006), p. 75 transl.
[143] cf. Kodukula (2006), p. 63 f.

4 Real Options in Petroleum Exploration and Production

4.1 Risk Modeling

Taking a look at the current crude oil prices it is obvious that especially in the last years the oil price was always around $ 100 per barrel. In particular the rapid growing emerging markets like China need a high demand of petroleum and seem to guarantee a high price even for the next decades.

This high energy demand drives the E&P industry to explore new opportunities and develop new technologies to be able to produce more petroleum from already existing mature fields or to unlock new resources. As a consequence the projects often incorporate high technical challenges and various uncertainties. An improvement of the decision making process is highly appreciated by the industry and a proper decision analysis can be an advantage on the market.

E&P projects are typically capital intensive and the long lasting projects are embedded in a risky environment where new information (e.g. basin history, increased understanding of formation properties) may become available over time. The simple net present value fails to react to new information. Hence decision tree analysis and real options valuations are becoming more popular.

The two main risks the E&P industry is facing are the same as for any other industry with the exception that both can be reasonably high. These two are the technical or private risks, that should be mitigated by a portfolio and the market risks that will affect the whole portfolio. Because in the E&P industry the market has a large volatility the possible number of outcomes is large whereas technical risks typically only have a limited number of outcomes. A few examples of technical risks are stuck pipe, geological uncertainties or corrosion in contrast to market risks like oil prices, political risks and rig rates.

Decision trees offer another quite popular possibility to address technical risks. The critical inputs are the probabilities and the cost estimations. To adequately evaluate market risks it is necessary to use the real options approach (Fig. 4-1). A static risk premium like in case of the net present value cannot compute the value of flexibility incorporated in any project[144].

[144] URL: http://www.palantirsolutions.com/news-and-events/research/presentations/modelling-in-the-e-p-industry-the-value-of-flexibi (08.08.2014)

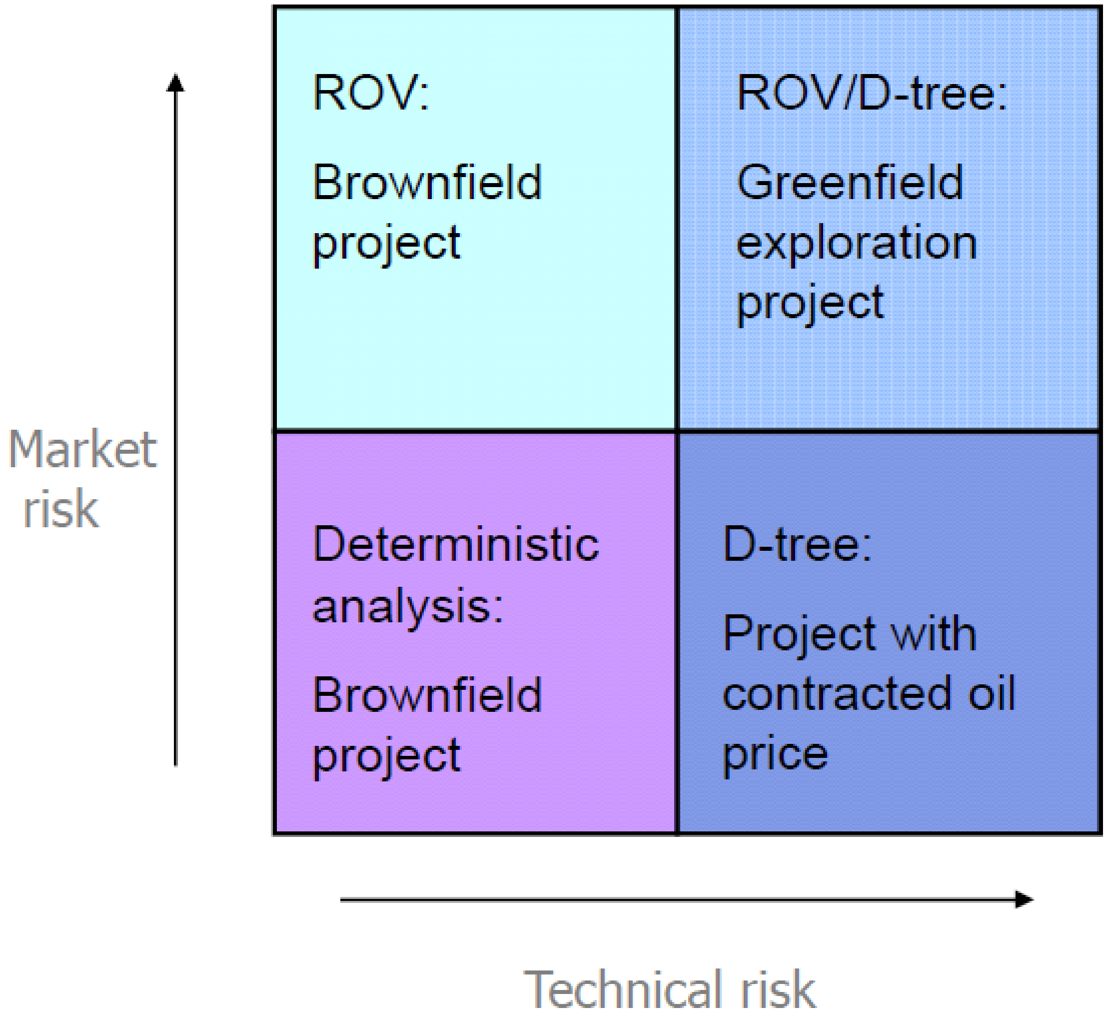

Fig. 4-1: Some E&P related projects and the recommended evaluation tool[145].

[145] URL: http://www.palantirsolutions.com/news-and-events/research/presentations/modelling-in-the-e-p-industry-the-value-of-flexibi (08.08.2014)

4.2 Real Option Analysis in E&P

After Stewart Myer introduced the term real options in 1977 many economists picked up his ideas and emphasized the benefits and powers of this new evaluation tool. However, the classical discounted cash flow analysis is still dominating the oil and gas industry.

Due to various assumptions and conditions and the vast number of different methods and types that are available decision makers might be overwhelmed. Moreover real options are not a replacement for the classical approach but a logical advancement and are much more time-consuming. In addition the final result might even be close to the net present value. Especially if the value of flexibility is low real options are not be best evaluation tools. Therefore it is of particular importance that decision makers are aware of the principle and the limitations of real options.

Particularly oil companies need to face an extreme uncertain environment every day - not only because of the volatile oil and gas price but also because of political and technical issues. The overall investments are huge and so are the sunk costs. The projects itself might last for a very long time and depending on the circumstances payback usually is rather late.

A discounted cash flow analysis under such conditions will most likely underestimate the economic value because uncertainty always has a negative impact on the net present value. In case of higher uncertainties the risk premium will increase. As a result the positive cash flows that are discounted from the future to the present day will be less and the project might be rejected. Besides management often has the possibility to actively interfere in the course of a project and change circumstances to their favor. This cannot be properly reflected by traditional evaluation tools. Higher uncertainty means a larger range of possible outcomes and hence creating high potential. The real options approach tries to mimic the real world and estimates the appropriate economic value[146].

One of the first workshops held by the Society of Petroleum Engineers that investigated the use of real options in the E&P industry were held in 2000 and 2002. An essential outcome was the so called "Banff taxonomy". The main objective was to clarify terms and clear up misconceptions (Fig. 4-2). The different approaches were analyzed and evaluated in a dynamic environment. Essential elements of the ranking process were flexibility, uncertainty and data input[147].

[146] cf. Jafarizadeh et al. (2009), p. 3 f.
[147] cf. Bratvold (2005), p. 2 f.

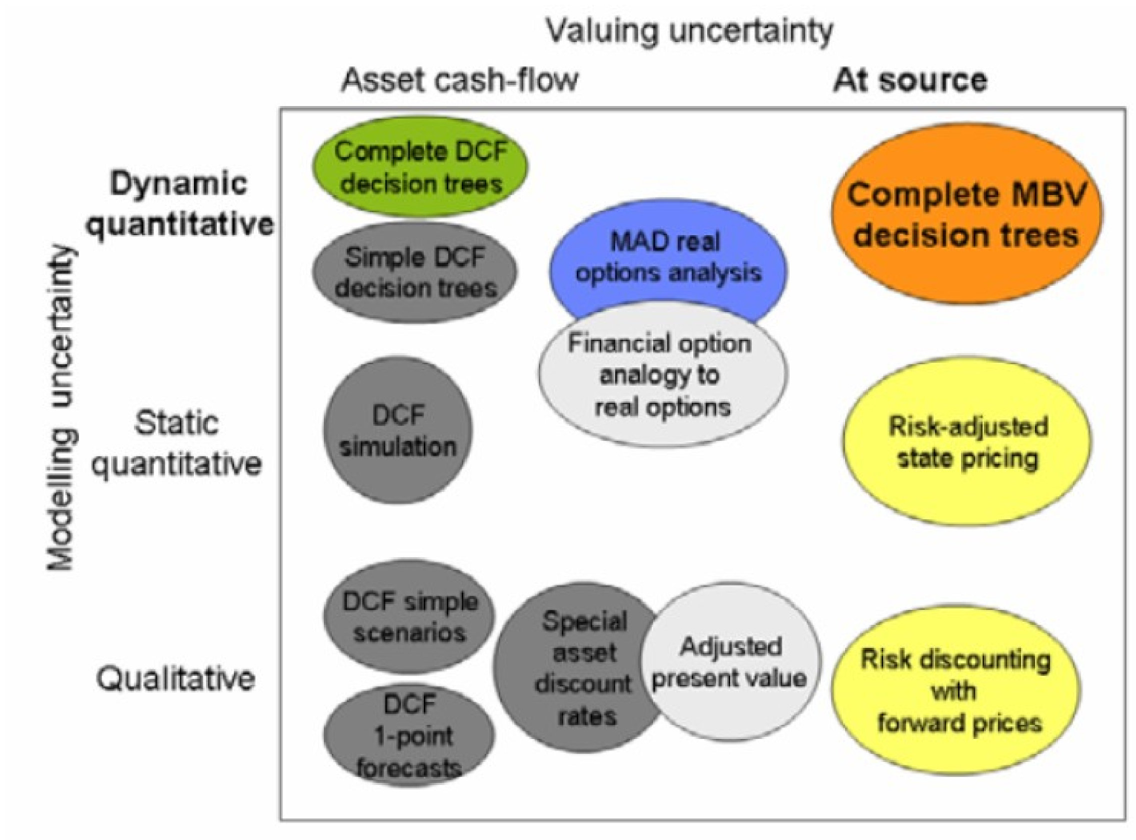

Fig. 4-2: The Banff taxonomy as the main output of workshops held in 2000 and 2002[148].

Moreover to make good decision it is important to have a logical and systematic way that accounts for a constantly changing world. Such a dynamic model needs to consist of parameters that are representative for that change. The classical real option approach is based on market values in contrast to fixed values like the risk premium in the traditional evaluations. The mathematical background of financial options is used for real assets. For E&P projects most of the conditions and requirements of the model are already met. E. g. only the owner of the license has the right to perform or not perform specific actions. This is usually the operated oil company that decides the course of actions. In case there is no uncertainty at all, the flexibility becomes zero and the initial economic value will remain constant over the life cycle of the project. Real options will add no additional value and the traditional tools are sufficient to evaluate the project. On the other hand if uncertainties are high the resulting high flexibilities may even prevent unwanted outcomes. Therefore real options don't only emphasis the upside aspects but also minimize the negative risks. The aim is to have a tool at hand that might need to be optimized for a given situation but is less influenced by optimism or pessimism of decision makers[149 150 151].

4.2.1 Typical Flexibilities and Types of Options

The circumstances under which is project is development as well as the intrinsic complexity defines the flexibility and the number of possible outcomes. The more opportunities the

[148] cf. Bratvold (2005), p. 2
[149] cf. Begg (2012), p.32 ff.
[150] cf. Jafarizadeh et al. (2012), p. 158 f.
[151] cf. Bratvold (2005), p. 1 ff.

more flexible a project is. Most commonly for an oil field development is the wait to invest opportunity. The major influence on the economic value of this type of option is the benefit of getting new information during the waiting period. A valid reason to wait could be a decrease in associated costs due to new materials, a new and faster way of drilling, a change in law and regulations, higher hydrocarbon prices or any other opportunity to increase revenues.

The traditional discounted cash flow analysis is characterized by an economic value based on a static discount rate and a possible price scenario. If the project has a positive NPV it will be carried out otherwise the company will most likely leave the prospect. According to the contract and the stated commitments oil companies usually have the opportunity to postpone their investment to a later time and this rescheduling has a specific positive economic impact. But the option to wait is not restricted to a unique field development phase, it can be carried out at every stage and hence can influence every single tollgate decision.

Another important and frequently evaluated flexibility in the E&P industry is the termination flexibility. In case there are no binding regulations or contract terms the optimal life of a field is constantly changing. Sometimes the expected lifetime at farm in can differ from the real optimal life of the field. For example because of new technologies or oil prices the economical field life might in reality be sooner or later than expected. To prevent losses on the one hand but achieve highest profit on the other hand the incorporated termination flexibility needs to be determined using an option to abandon.

In the real world it might even be economic to shut down the project temporarily. If the operating costs are for example higher than the revenues the company may temporarily shut down the project and look for more favorable circumstances. Because of the accumulating start and stop costs this might not be an option during the production phase but can be a reasonable consideration in other stages.

More sophisticated options are needed to evaluate operational flexibilities which represent another important option type in the petroleum exploration and production. There are a vast number of operational flexibilities and therefore these options are usually classified into two main groups. The first group comprises of options because of performed activities. An example would be a seismic survey to increase the likelihood to find oil or gas. Another one could be the number of wells, the wellpath or the total depth of the wells that can significantly influence the production profile. Whereas the second group consists of options that create value due to changing the project duration. Because of the time value concept of money an early production is always favorable but can be further restricted due to regulations or better opportunities in a nearby block using the already existing facilities.

The presented flexibilities are only the most common ones. Especially the for a specific project designed options can exploit the maximum flexibility and result in a high option value. Fig. 4-3 summarizes the most frequent and already discussed opportunities that are typically considered in E&P projects[152].

[152] cf. Jafarizadeh et al. (2009), p. 2 f.

Categories		Examples
Wait-to-Invest Flexibility		Defer Investment; Defer the Decision to carry the Project to the next Stage
Termination Flexibility		Abandonment Decision
		Contingencies to Account for Legislative Changes
Temporary Start/Stop Flexibility		Defer the decision to develop a Field, Temporary Shut-down of Production
Operational Flexibility	Activity Level	Number of wells, Depth of wells
		Platform capacity design
		Information Acquisition/learning Options
		Growth Opportunities
		Upcoming Opportunities in Nearby Fields
		Conversion of Production Wells into Injection Wells
		Farm-in or Farm-out?
		Well Maintenance Programs
	Operational period	Change in Production/depletion Rate

Fig. 4-3: Summary of the most common E&P flexibilities[153].

4.2.2 The Market Asset Disclaimer Approach

Most managers in E&P are aware of the upside potential of uncertainties and hence ignore the results of the traditional cash flow analysis. They point out the high flexibility and strategic value but on the other hand cannot give any estimate of the incorporated chances.

This dilemma already highlights the need for a new approach that can mimic the dynamic environment. Real options are the logical advancement.

There are quite some different real option approaches discussed in the literature but only a few of them were ever considered for projects. Moreover real options are not only a valuation technique but a way of thinking. It is all about chances and flexibilities that can hedge projects against unwanted outcomes.

Assume a call option that gives the holder the right to acquire a stock for a fixed predetermined price. The option writer hedged the risk of falling prices and till the option expires the holder is able to exercise the option and buy the stock for a specific price. On the other hand a company that acquired a license has the right to develop that field within a certain period of time. According to the circumstances there might be some reasons to wait, e.g. for higher oil prices or till some uncertainties are solved.

The most basic foundation is that the market is at equilibrium. If this condition is met the prices can serve as a benchmark for any asset. Many experts argue that markets are most likely at equilibrium and no one should be able to beat the market in a long term[154].

However, there might be two relevant systematic errors if the same equations for financial options are used to evaluate real assets in E&P.

[153] cf. Jafarizadeh et al. (2009), p. 4
[154] cf. Jafarizadeh et al. (2009), p. 7 f.

First of all according to Black and Scholes the option value follows a geometric Brownian motion. While this is generally the case for stocks the project value in the E&P industry is often controlled by the oil price. The oil price on the other hand rather seems to follow a mean reverting process instead of a geometric Brownian motion. Hence, Black and Scholes as well as the binomial approach needs to be used with caution.

The second assumption is the existence of a perfect replicating portfolio that is highly questionable. Black and Scholes established a formula to calculate the value of European options. It was assumed that two stocks with the same cash flow structure and the same risks have the same price in perfect, arbitrage free markets. So if we could find a portfolio of stocks with the same pattern as our real asset the price of the portfolio would be the same as the one for the real asset. Furthermore it implicates that real assets can be traded on the market. Finding a similar asset in the financial markets is not trivial and approaches were developed that don't rely on information about the financial market. Consequently the input parameters that were gathered directly from the market were then exchanged for information offered by experts and experience. The resulting combination of subjective and market data and the inconsistency led to the development of the market asset disclaimer approach[155].

Several approaches were developed that rely solely on expert data to further detach from the initial market price estimation according to the replicating portfolio. Copeland & Antikarov (2001) proposed the so called market asset disclaimer. In this model the underlying parameters can be determined with the NPV criterion. The net present value of the project without considering any flexibility is according to Copeland & Antikarov (2001) the best proxy of the market value as if it were traded[156]. This approach totally ignores any market data of the replicating portfolio and uses instead a classical NPV calculation.

They argue that even the NPV is based on the assumption of traded assets of comparable risks. Imagine the determination of the weighted average cost of capital where the market based betas were used to estimate the interest rate of the equity.

So if the NPV represents the value of a traded inflexible asset then the real option value as an addon to the NPV can determine the value of the flexible investment as if it would be traded on the market. Experts like Copeland & Antikarov (2001) as well as Trigeorgis (1996)[157] believe that this is a valid approach that can accurately estimate the value of the project`s flexibility.

Another assumption that is usually made, is that the asset prices follow a geometric Brownian motion. Finally this leads to the conclusion that MAD cannot only be used for projects where prices can be directly observed on the capital market like oil, gas and gold but also for more general investment decisions.

Furthermore the market asset disclaimer approach is far easier to apply from a management perspective and is not too restricted. The method is typically divided into five steps:

1. calculate the net revenues and corresponding cash flows for each year
2. discount the cash flows to the present and sum it up to get the net present value
3. as a discount rate use the WACC instead of any other market established rates
4. estimate the uncertainties related to the inputs (e.g. apply a Monte Carlo Simulation)

155 cf. Borison (2005), p. 17 f.
156 cf. Copeland et al. (2001), p. 93 f.
157 cf. Trigerogis (1996), p. 121 ff.

5. build a risk neutral binomial tree (random walk hypothesis) according to the estimated uncertainties and determine the real option value in a role back algorithm

Even if the option and the underlying asset are priced consistently, this approach is completely independent of the capital market. Therefore it can happen that the prices are too high or too low compared to the real market values. Moreover the approach is built on many subjective inputs.

The method is only recommended if no market analogous exist or there are no observable market prices available[158].

4.2.3 The Integrated Approach

The integrated approach is especially designed for various kinds of uncertainties. Many projects can be affected by market risks like oil prices as well as private risks like HR related topics.

In case of complete markets where the information from the markets can be used to derive an optimal decision, the real options value should only be based on the market itself. On the other hand in a more realistic situation markets might not be complete and projects face a wide range of technical uncertainties. These private uncertainties are not valued in capital markets and hence need to be considered in a separate phase.

Therefore an integrated approach was developed that is built on the following six steps:

1. establish a decision tree with the investment alternatives
2. calculate the incorporated uncertainties and classify them as either public or private
3. for the public or market uncertainties indentify an appropriate portfolio with the same cash flow structure and estimate probabilities
4. at each end point of the decision tree determine the NPV
5. use the risk free rate as the discount rate
6. pick the optimal strategy and calculate the associated economic value using the role back algorithm

In contrast to the classical methods like Black and Scholes or MAD this approach can incorporate both types of uncertainties. The price risks are correctly estimated using market information whereas for example geological uncertainties in the exploration phase of the project can be modeled using expert opinions.

4.2.4 A Comparison

In principle the two presented techniques - the market asset disclaimer and the integrated approach - are the same under a complete market. A perfect replicating portfolio would estimate the same value as a DCF calculation. Not only is it difficult to find a traded asset for a non-traded asset that mimics the cash flow and risk structure it is also tricky to determine market uncertainties. For example it might be questionable if the replicating portfolio is capable to reproduce the volatility of the oil price. Especially if projects are complex an appropriate portfolio can be very difficult to find on the market. The MAD approach is a good alternative then.

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[158] cf. Jafarizadeh et al. (2009), p. 7 f.

5 Case Study OMV

First of all the current state of project management and established evaluation techniques at OMV will be defined. The first chapter describes the key principles and indicators used in project management. The essential maturation path with the five distinct project phases is described and the main activities along that path are presented. After a short introduction the standardized project evaluation will be discussed. To achieve the financial targets OMV defined a five-step closed loop as their main management toll for the evaluation of projects. The internal rate of return as well as the payback period are the most fundamental key performance indicators at OMV. The profitability index on the other hand is used for ranking projects.

In the second part of the chapter some numerical examples of real options are presented and compared to the traditional discounted cash flow analysis. Moreover the six step binomial lattice approach and the analytical Black and Scholes method are demonstrated and discussed.

5.1 Current State OMV

5.1.1 Project Management

In 2010 OMV implemented standardized key principles and practices in capital project management to increase the effectiveness and efficiency in the management of projects with a total budget above not only but especially € 20 million. For this purpose OMV introduced best practices and harmonized working methods across the whole OMV Group to improve the overall performance[159].

5.1.2 The Six Principles

Six key principles form the basis of the capital project management at OMV.

1.) **The Maturation Path**

A definite maturation path with five individual, consecutive project phases and their tollgates is the first key principle. In detail these are “Identify and Assess”, “Select”, “Define”, “Execute” and “Operate”.

Fig. 5-1: The maturation path with five individual, consecutive project phases “Identify and Assess”, “Select”, “Define”, “Execute” and “Operate” as well as their tollgates presented in green diamonds[160].

The first phase consists of the identification of the opportunity itself. “Identify and Assess” ends if there is a concept that likely delivers success in most external outcomes (= tollgate 1). At this stage the project enters the “Select” phase in which a final proposal is chose and endorsed. After the tollgate 2 the project enters the “Define” phase. Now all project specifications are considered and a final project approval is made (= tollgate 3). Afterwards the project is realized and handed over to the asset owner. Finally during “Operate” steady-state operations are achieved and lessons-learned are captured. Fig. 5-2 represents the overall process and the final decisions of every individual project phase as well as the required activities concerning the project team.

[159] cf. Danzinger (2011), p. 1
[160] cf. Danzinger (2011), p. 2

Decision	1: Have we looked wide enough? Is there at least one Option that will work in most, or all, Outcomes?	2: Have we selected the best Concept?	3: Is everything in place to ensure success?	4: Is it safe to energize or introduce hydrocarbons? Can the Asset Owner take on the (additional) operational accountability?	5: Have we delivered the promise? What can we learn for future projects and how do we ensure lessons are learned?
Steer from Project Owner	Initiate Opportunity and Assess the range of possible Options and (external) Outcomes	Make all concept-defining and strategic decisions	Draw up the specification(s) for the Project. Prepare for Project Sanction	Deliver the promise made during Project Sanction	Establish stable Operations, evaluate the Business Case, capture lessons learned and disband the team
Main Activities of the Project Team	"Frame" the Opportunity: Purpose, Perspective, and Scope. Articulate the Business Case (in alignment with invest Controlling), align with corporate strategy and resource the team. Assess own Options and range of external Outcomes	Select the best alternative to deliver value. Set all key technical and commercial strategies. Make trade-offs between recognised value drivers, if applicable	Define the selected alternative into a final Project Specification. Invite bids and obtain critical materials or equipment with long lead-times. Obtain permits and prepare Project Sanction. Align the Business case with invest Controlling and ensure the necessary approvals are in place	Award to selected bidders. Ensure detail design, procurement and construction is pursued safely, along approved standards and specifications	Assist the Asset Owner with Commissioning the facilities and establishing stable operations. Collect lessons learned and disband the team

Fig. 5-2: The five individual project phases and their final decisions to enter the consecutive phase as well as the essential activities of the project team[161].

2.) **Project Organization and Governance**

The temporary project organization is clearly defined and definitely linked to the permanent company line organization through a Senior Vice President who appoints the project owner and is supported by a steering committee. The project owner is single point accountable for every aspect of the project and can authorize to evolve along the maturation path but usually the financial, commercial or organizational authority lies within the organizational positions.

The project team established during the phase "Identify and Assess" consists of members of different disciplines but at least of an Asset Development Manager, Project Manager and Asset Representative (Fig. 5-3). An active dialogue between responsible management and the project team should be in place. Choices should be decision driven instead of activity driven and every member contributes to the success[162].

[161] cf. Danzinger (2011), p. 3
[162] cf. Danzinger (2011), p. 4 f.

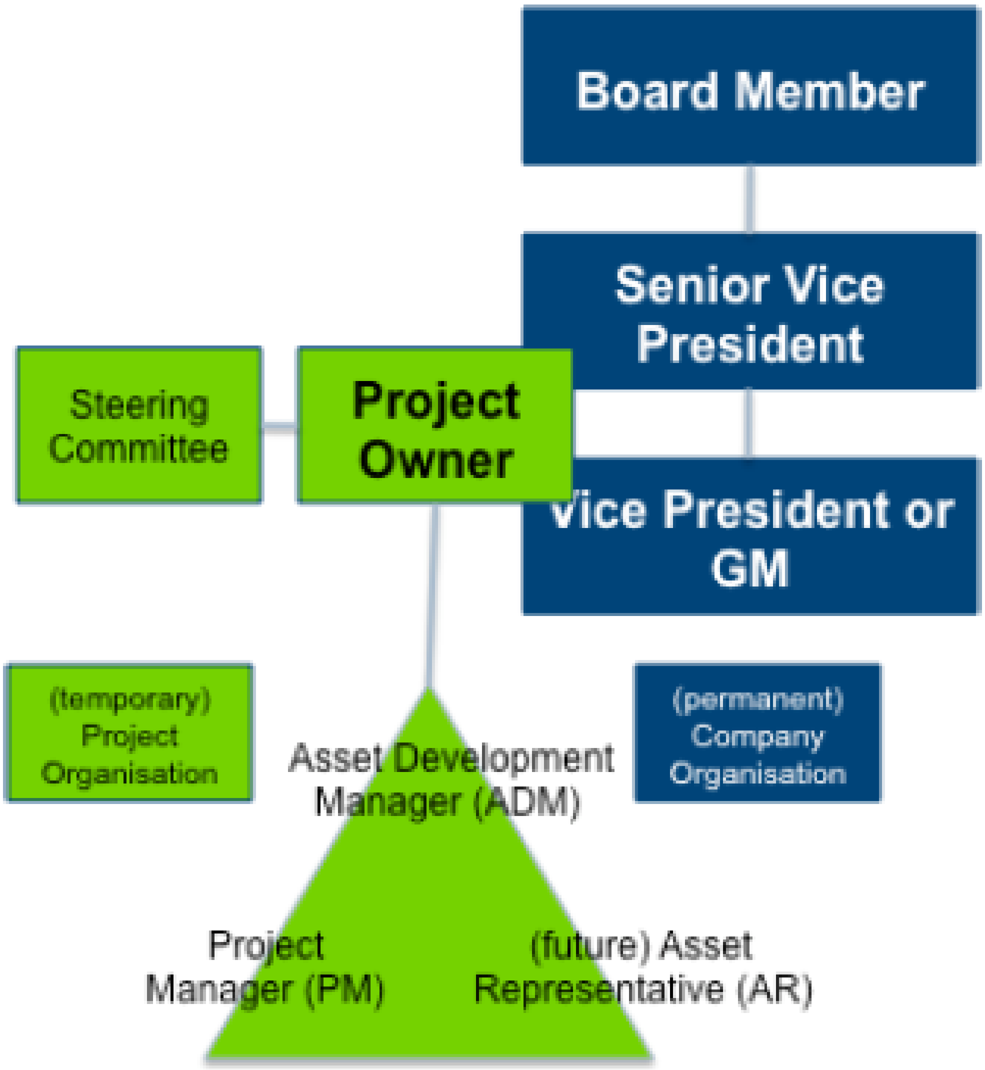

Fig. 5-3: The temporary project organisation and the link through the senior vice president to the permanent company organisation[163].

3.) **Risk Management**

Capital project management has the task to manage project risks which are defined as possible events with positive or negative outcome.

A project assurance plan including so called value improving practices (VIP) needs to be provided by the capital project management to increase the stakeholder's confidence. These VIPs originally developed by IPA[164] one of the leading companies in project evaluation is a quantitative analysis of the management system in place. In general there are twelve distinct VIPs such as technology selection, process simplification, waste minimization, energy optimization, predictive maintenance or benchmarking. These documented repeatable work procedures with measurable results can help to improve a project`s performance. Best practices that need to be considered are opportunity farming, front end loading (FEL), independent gate reviews (IGR), conflict resolution and the implementation of lessons

[163] cf. Danzinger (2011), p. 2
[164] URL: http://www.ipaglobal.com/ (28.12.2013)

learned. At least at the beginning of a project phase and after a major change the project team performs a brainstorming and summarizes the results in a high-level plan called "Roadmap" that acts as a projects lifecycle. Front end loading on the other hand emphasis the need for robust planning and design in an early stage of the lifecycle. It has become apparent that the additional costs for the FEL are insignificant to any other alternative at a later phase. Independent gate reviews are reviews by independent experts before a tollgate. In case of conflicts e.g. between the project owner and the steering committee the best practice according to the conflict resolution is to move one level up in the organizational hierarchy. Finally all lessons learned should be well documented at every tollgate and if possible stored and distributed.

4.) **Provide Project Assurance Plan**

Project managers need to establish an assurance plan to provide confidence to stakeholders.

5.) **Project Resources**

The capital project management must ensure that at every stage the project has access to money, manpower and other needed resources.

6.) **Framework to promote an Interdisciplinary Approach**

Capital projects should have a framework that encourages an interdisciplinary approach[165].

5.1.3 Project Evaluation

An authentic and standardized evaluation of projects in the E&P business is crucial. This process is built on high quality decisions which require transparent information, data processing and application procedures. In order to achieve the financial targets an iterative five-step closed loop management method similar to the Deming cycle[166] is in place to control and improve the evaluation practice. This process consists of the five steps planning, information, qualitative and quantitative decision-making, approving and controlling[167].

According to the corporate directive the following requirements need to be met before the company approves an investment:

- compliance with strategic goals
- increase earnings (achieving or exceeding financials goals)
- increase shareholder value[168]

The internal rate of return as one of the most important key performance indicators acts as the primary selection criterion and needs to meet or exceed the following rates:

for E&P

- organic growth 15%
- acquisitions 11%

for G&P

- regulated business (gas transmission & storage) 8%
- non-regulated business 12%

[165] cf. Danzinger (2011), p. 3 f.
[166] http://en.wikipedia.org/wiki/W._Edwards_Deming (25.12.2013)
[167] cf. Sieh & Schleicher (2012), p.1 f.
[168] cf. Sieh & Schleicher (2012), p. 7 f.

for R&M

- in general 15%

for C&O

- in general 13%[169]

The payback period as the time required to recoup the funds ("pay for itself") is at OMV the second evaluation criterion. This method does not take the time value of money into account. The payback period needs to be less than 10 years regardless of the type of project.

In a second step the projects will be ranked according to the profitability index as the ratio of the net present value and the discounted CAPEX. The NPV is calculated using a predefined after tax discount rate. As of 2012 these discount rates are:

- for E&P 10,0%
- for G&P 7,0%
- for R&M 7,5%
- for C&O 8,0%[170]

All presented discount rates are considered to be at least close to the weighted average cost of capital (= WACC) as the rate OMV needs to pay on average to all providers of capital.

For every project within OMV a risk analysis including financial, socio-economic and environmental concerns is mandatory. As part of the risk analysis the different risks are defined and the probability of occurrence as well as the impact is evaluated. Furthermore issues are addressed to prevent or decrease essential risks for the company. For investments above € 20 million the best and the worst case scenario regarding the economics are determined.[171]

5.1.4 Real Options

Real options are not yet used in the standardized process for the evaluation of projects at OMV. To the authors knowledge two thesis, one PhD and one Master thesis were already written upon this topic. Both papers present the theory behind real options and give examples. The PhD thesis especially describes an advanced tool to evaluate projects in the oil & gas business. The author implemented a second level binomial tree to incorporate all available information to improve estimates.

Moreover the real options approach is perhaps of the well known hurdles not yet a standardized tool in the evaluation process at OMV.

[169] cf. Sieh & Schleicher (2012), p. 8
[170] cf. Sieh & Schleicher (2012), p. 9
[171] cf. Sieh & Schleicher (2012), p. 9

5.2 OMV West Africa: Development of a Deep Water Oil Field

To find the petroleum in an underground structure you need to invest in exploration even if the extraction turns out to be not economic. These are so called sunk costs and are a requirement to use real options.

The profitability of the field itself depends on the future prices, the amount of petroleum the company is able to recover and taxes as well as royalties. These parameters can be quite uncertain at the beginning but if the investment is made and operations are carried out, more information will become available and some uncertainties might clear. This would as well have an impact on the risk premium in the DCF but because the discount rate is static it cannot be considered. Therefore DCF will constantly underestimate the value of the project. In real options the present value is based on market values and/or expert opinions and the riskless interest rate. Nevertheless technical also called private risks like issues with new technologies or HR related topics are not considered in the classical approach[172]. Only systematic, market based risks can be evaluated in the classical ROA like the analytical method from Black and Scholes.

Lets imagine OMV has discovered and developed a deep water oil field offshore West Africa. The company is the owner of the license and would like to drill another production well within the next five years to increase production to be at maximum throughput. Because this well is part of a commitment it needs to be drilled within the given time or the company has to abandon the field. Even if the production profile is uncertain the well will come on stream in the same year. The variable production costs of $ 10 per barrel will not change over the life time of the field.

Furthermore different price forecasts and production profiles are considered and used to evaluate the impact on the economic value. For the best and worst case scenario with a probability of 25 % the following two production figures for this well are assumed (tab. 5-1).

[172] cf. Kodukula (2006), p. 5 f.

Table 5-1: Daily production forecast for a well drilled offshore West Africa.

year	daily production 1 [bbl/day]	daily production 2 [bbl/day]
1997	0	0
1998	4500	4950
1999	4000	4400
2000	2500	2750
2001	2000	2200
2002	1500	1650
2003	1250	1375
2004	1000	1100
2005	900	990
2006	800	880
2007	750	825
2008	725	798
2009	650	715
2010	600	660
2011	500	550
2012	250	275
	worst case	**best case**

Because of maintenance and possible technical concerns the total uptime of the well is thought to be 350 days per year. The calculated production each year is presented in Fig. 5-4. The maximum field life time is 15 years from now and the drilling as well as the additional slot on the platform are estimated to cost $ 300 million independent of time. In addition this well will increase the field costs by a fixed amount of $ 8 million per year.

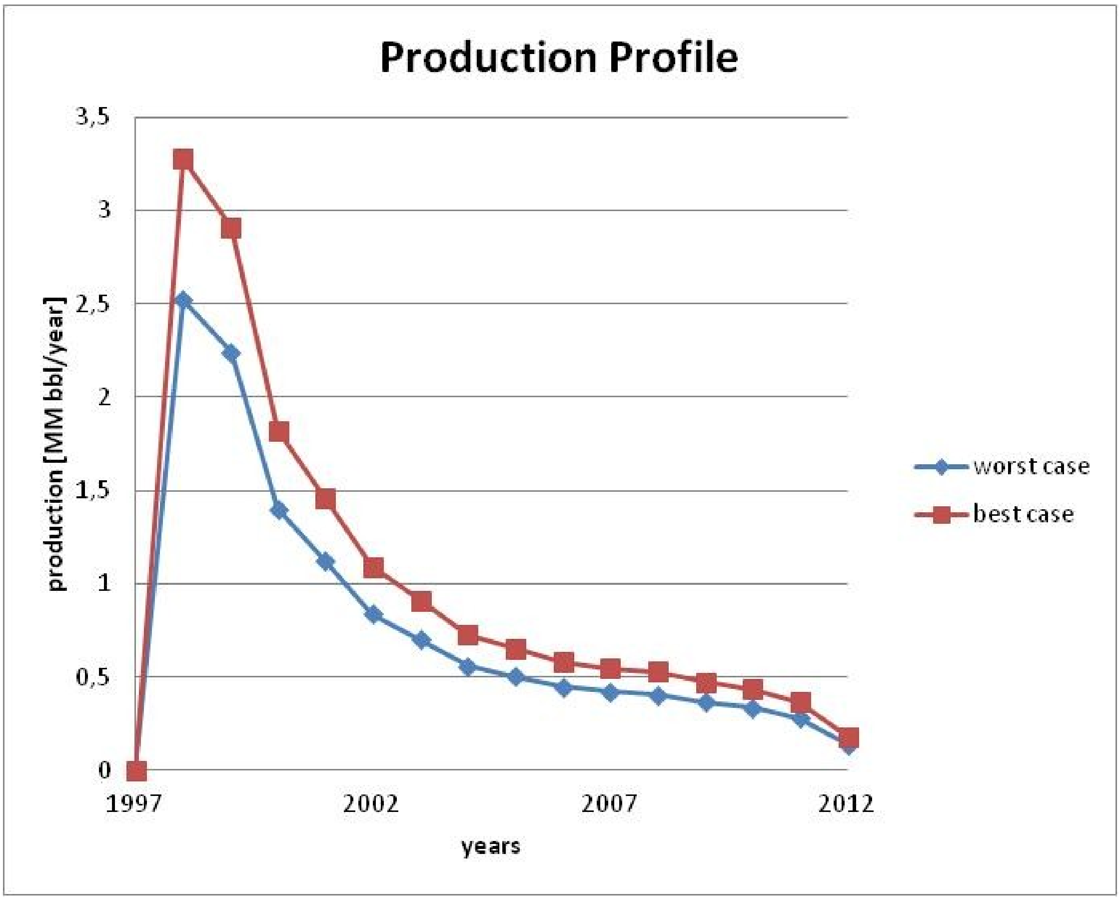

Fig. 5-4: The estimated production for each year over the whole life cycle of fifteen years.

5.2.1 The Traditional Approach: Discounted Cash Flow Analysis

The DCF is only a now or never decision and cannot value the possibility to delay drilling by a maximum of five years in this example. The cash flow for each year was calculated and the net present value was estimated using two different pricing scenarios. The first assumes a constant oil price of $ 80 per barrel whereas the second one is based on historical data from 1997 to 2012 representing the average spot market price of Brent (Fig. 5-5). In each scenario the well was drilled in the first year and will come on stream within the same year. The discount rate is according to OMV standard procedure fixed at 10 %.

Average annual spot crude oil prices, $US per barrel

	Cash prices	2012 dollars
1861	0.49	12.47
1870	3.86	69.80
1880	0.95	22.51
1890	0.87	22.14
1900	1.19	32.71
1910	0.61	14.97
1920	3.07	35.17
1930	1.19	16.35
1940	1.02	16.69
1950	1.71	16.30
1960	1.90	14.71
1970	1.80	10.64
1980	36.83	102.62
1990	23.73	41.68
1991	20.00	33.72
1992	19.32	31.62
1993	16.97	26.97
1994	15.82	24.50
1995	17.02	25.64
1996	20.67	30.24
1997	19.09	27.31
1998	12.72	17.91
1999	17.97	24.76
2000	28.50	37.99
2001	24.44	31.69
2002	25.02	31.94
2003	28.83	35.97
2004	38.27	46.51
2005	54.52	64.09
2006	65.14	74.19
2007	72.39	80.16
2008	97.26	103.71
2009	61.67	66.00
2010	79.50	83.70
2011	111.26	113.56
2012	111.67	111.67

Fig. 5-5: Crude oil spot prices from 1861 to 2012[173].

The results of the net present value are presented in Table 5-2. The NPV suggests that only in case of the constant price level and the best case production scenario the investment should be made. The historical prices are most likely not appropriate and the resulting NPV is highly negative. If the net present value is close to zero it is often difficult to make a decision.

[173] cf. BP (2011), p. 6

Table 5-2: Calculation of the net present value considering the given cost profile and the discount rate of 10%. 1A and 2A correspond to the worst case production scenario whereas 1B and 2B consider the production according to the best case scenario.

year	price [$/bbl]	net revenue 1A [$]	net revenue 1B [$]	price 2 [$/bbl]	net revenue 2A [$]	net revenue 2B [$]
1997	80	-300000000	-300000000	27,31	-300000000	-300000000
1998	80	102250000	113275000	17,91	4458250	5704075
1999	80	90000000	99800000	24,76	12664000	14730400
2000	80	53250000	59375000	37,99	16491250	18940375
2001	80	41000000	45900000	31,69	7183000	8701300
2002	80	28750000	32425000	31,94	3518500	4670350
2003	80	22625000	25687500	35,97	3361875	4498062,5
2004	80	16500000	18950000	46,51	4778500	6056350
2005	80	14050000	16255000	64,09	9038350	10742185
2006	80	11600000	13560000	74,19	9973200	11770520
2007	80	10375000	12212500	80,16	10417000	12258700
2008	80	9762500	11538750	103,71	15778912,5	18156803,75
2009	80	7925000	9517500	66,00	4740000	6014000
2010	80	6700000	8170000	83,70	7477000	9024700
2011	80	4250000	5475000	113,56	10123000	11935300
2012	80	-1875000	-1262500	111,67	896125	1785737,5
NPV [MM $]:		**-2**	**34**		**-237**	**-225**

5.2.2 The Binomial Lattice in Six Steps

In contrast to the NPV the binomial lattice approach in combination with the market asset disclaimer (= MAD) which does not further rely on the replicating portfolio or the integrated approach which is additionally based on experience can be applied. As a starting point the approach uses the net present value without considering any flexibility. Copeland & Antikarov 2001 argue that even in case of the market asset disclaimer appraoch the NPV is in this sense the best unbiased estimate of the market value of the project. The net present value will be projected to the future according to a binomial lattice. At each node an up or a down movement according to the proportional standard deviation of the DCF or if market based the risk free interest rate is calculated. After constructing the lattice the option value will be estimated using a role back algorithm[174].

As a first step the underlying asset value needs to be determined. This is simply done by calculating the net present value without considering any managerial flexibility. Bratvold et al. (2005) bring forward the argument that this first phase might be the weak point of the analysis. Every further step is based on the classical net present value and even if market parameters like the volatility are used for the simulation this add-on to the static discounted cash flow analysis may not be able to address complicated forms of flexibility. The main issues arise because of the fact that the major uncertain determinants of the cash flows are

[174] cf. Copeland et al. (2001), p. 21 ff.

not used directly in the calculation. Nevertheless uncertainties and flexibilities are modeled and a consistent valuation is guaranteed[175].

In the following phase a scenario tree is set up and the risk adjustments are determined using a binomial approximation to the geometric Brownian motion. Afterwards the option values are calculated similar to a decision tree analysis.

To consider technical uncertainties a second volatility would need to be implemented like in rainbow options. In this example the volatility is fixed at 25 %. The option to wait was selected as the main flexibility of the project at hand.

The construction of a binomial lattice is divided into six main steps. These are:

1.) **Frame the Application**

In the first step the main parameters are identified and described. The problem should be kept simple and easy.

2.) **State Input Parameters**

The second step involves a clear statement of the needed input parameters. In particular these are the underlying asset value, the strike price, the option life, the volatility factor, the interest rate and the time increment.

3.) **Define Option Parameters**

These parameters are necessary to determine the up and down movement.

4.) **Drawn the Binomial Tree and Calculate the Asset Values**

At each node of the created binomial tree the underlying asset value is estimated. This calculation starts at the left side and moves along the branches of the tree to the right.

5.) **Determine Option Values by Backward Induction**

Starting at the far right side the option value can be calculated by backward induction. Moreover the option value represents the maximum value of continuing or exercising the option. When reaching the left side of the tree at time zero, the final option value is determined.

6.) **Analyze the Results**

The estimated option value needs to be compared to the DCF or to other real option techniques to get a better understanding of the value of flexibility[176].

5.2.3 The Option to Wait

The option to wait is a very typical type for E&P projects. Especially if projects have a negative NPV but high uncertainty the company may want to wait till new information becomes available and uncertainties are cleared.

In the presented example the worst case production scenario with the constant crude oil price of $ 80 shows a net present value that is slightly negative and could in case of a reasonable uncertainty become positive.

Because the option to wait will be executed only if payoff is greater than the investment it can be compared with a financial call option. Since the acquired license of the oil field has a

[175] cf. Bratvold et al. (2005), p. 8 f.
[176] cf. Kodukula (2006), p. 97 ff.

distinct life the so called leakage will be considered. These potential losses of revenues are similar to the dividends of financial options and depending on the project are calculated using a constant or varying rate.

The already given input parameters for this example are:

- the underlying asset value of the worst case scenario with a constant price level which is according to the DCF $ 298 million resulting in a net present value of $ -2 million
- the time to expiration is five years
- the project volatility as a combination of the main project uncertainties was estimated to be 25 %
- the risk free interest rate is according to Fig. 2-3 close to 0.7 %
- the investment costs for the drilling and the slot on the platform are estimated to be $ 300 million
- the annual leakage rate is by comparison with similar projects around 5 %
- the time increment is set at one year which results in a valuation tree of five steps

The example will be calculated with and without the mentioned leakage factor as a constant value.

The second step incorporates the definition of the option parameters, which are representing the factors of the up and down movement. The up and down factors, u and d, are a function of volatility of the underlying asset value and the time increment:

$$(80)\ \mathrm{u} = \exp\ (\text{volatility} * \sqrt{\text{time increment}})$$

$$(81)\ \mathrm{d} = \exp\ (-\text{volatility} * \sqrt{\text{time increment}})$$

which can be rewritten to

$$(82)\ \mathrm{d} = \frac{1}{\mathrm{u}}$$

The risk-neutral probability, called p, is defined as:

$$(83)\ \mathrm{p} = \frac{\exp(\text{risk free interest rate} * \text{time increment}) - \mathrm{d}}{\mathrm{u} - \mathrm{d}}$$

This is a pseudo probability and is not the same as an objective probability used in a decision tree analysis. It will be used to weight the future option values.

Consequently for the project in West Africa the option parameters are:

$$\mathrm{u} = \exp\ (0.25 * \sqrt{1}) = 1.284$$

$$\mathrm{d} = \frac{1}{1.284} = 0.779$$

$$\mathrm{p} = \frac{1.007 - 0.779}{1.284 - 0.779} = 0.452$$

In case of leakage the risk neutral probability needs to be adjusted.

$$(84)\ \mathrm{p} = \frac{\exp((\text{risk free interest rate} - \text{leakage}) * \text{time increment}) - \mathrm{d}}{\mathrm{u} - \mathrm{d}}$$

The resulting neutral probability is then instead of 0.452

$$p = \frac{0.958 - 0.779}{1.284 - 0.779} = 0.354$$

Now it's time to build the binomial tree and calculate the asset value at each node of the tree. This is simple done by multiplying the underlying asset value with the corresponding up and down factor.

The next step is to calculate the actual options value at each node by backward induction. Starting at the far right side the value at the node is determined by subtracting the investment costs from the underlying asset value. This represents the investment at that point in time. At every node you have either the chance to invest or wait until the next year, except of the last year where you have to invest or leave the field. If the underlying asset value of a node is smaller than the investment needed to develop the resource the option will not be exercised and will expire. It is important to remember that an option is a right but not an obligation, so the options value cannot be negative (not considering the options premium or in other words any further commitments).

If you move one step to the left side to the first intermediate node the discounted weighted average of potential future option values using the risk neutral probability needs to be calculated. For example for the node "S*u*u*u*u" this would be

$$(85)\ \text{option value} = (p * S * u^5 + (1 - p) * S * u^4 * d) * \exp(-\text{risk free rate} * \text{time increment})$$

or

$$\text{option value} = (0.452*740+(1\text{-}0.452)*331)*\exp(\text{-}0.007) = 515\ \text{million}$$

In contrast when considering leakage the options value at this node is

$$\text{option value} = (0.354*740+(1\text{-}0.354)*331)*\exp(\text{-}0.007) = 474\ \text{million}$$

If the option would be exercised at this node the payoff is \$ 810 million minus the investment of \$ 300 million. Because the corresponding \$ 510 million is lower than the option value of \$ 515 million the company should not exercise the option but continue to wait.

In contrast at node "S*u*d*d*d" the option value to keep it open is zero. The payoff is only \$ 118 million which would result in a net loss of \$ 182 million. As a consequence the option will be kept instead of exercised (Fig. 5-6 & Fig. 5-7).

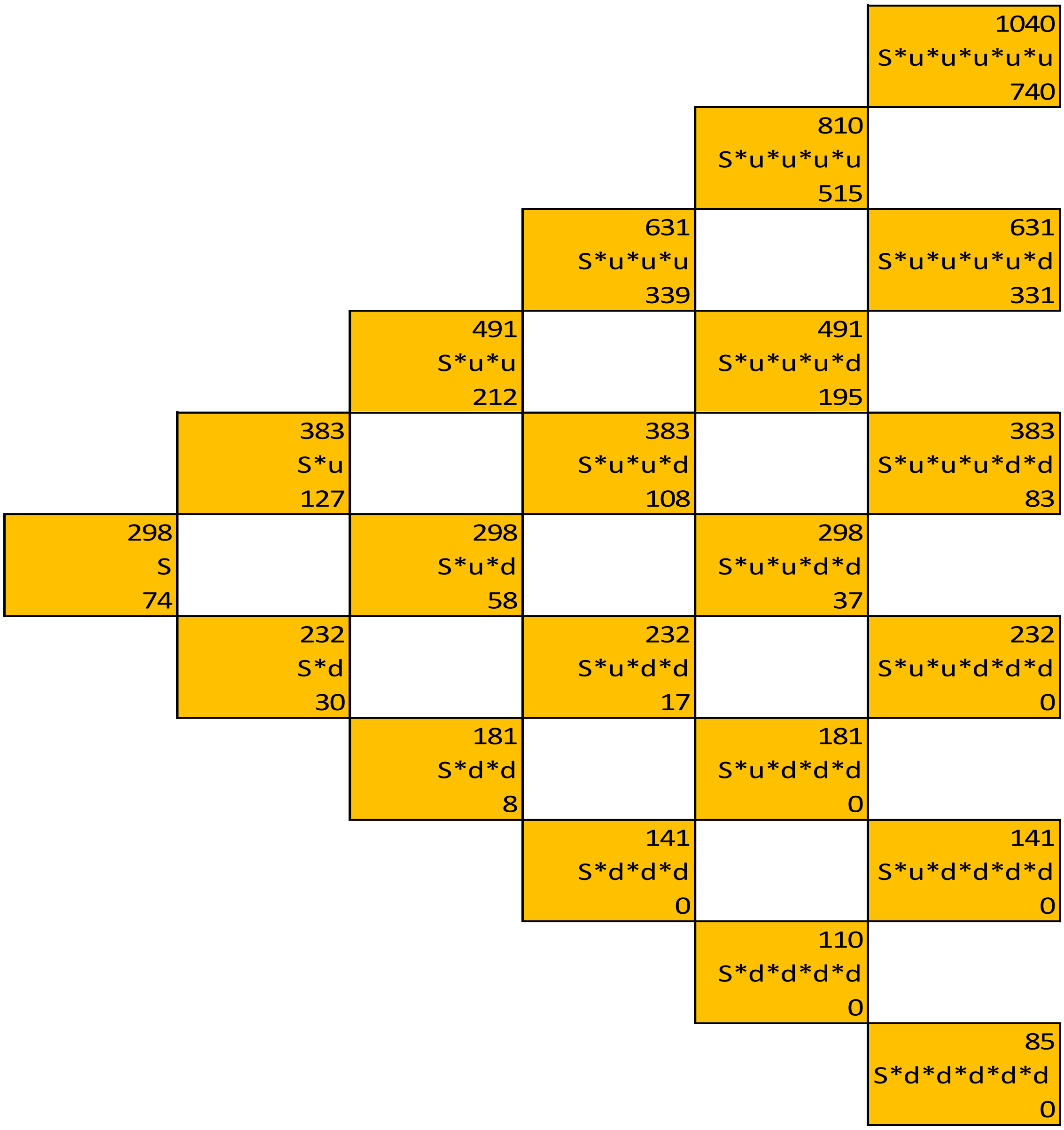

Fig. 5-6: The binomial lattice in case of no leakage (all values *10^6).

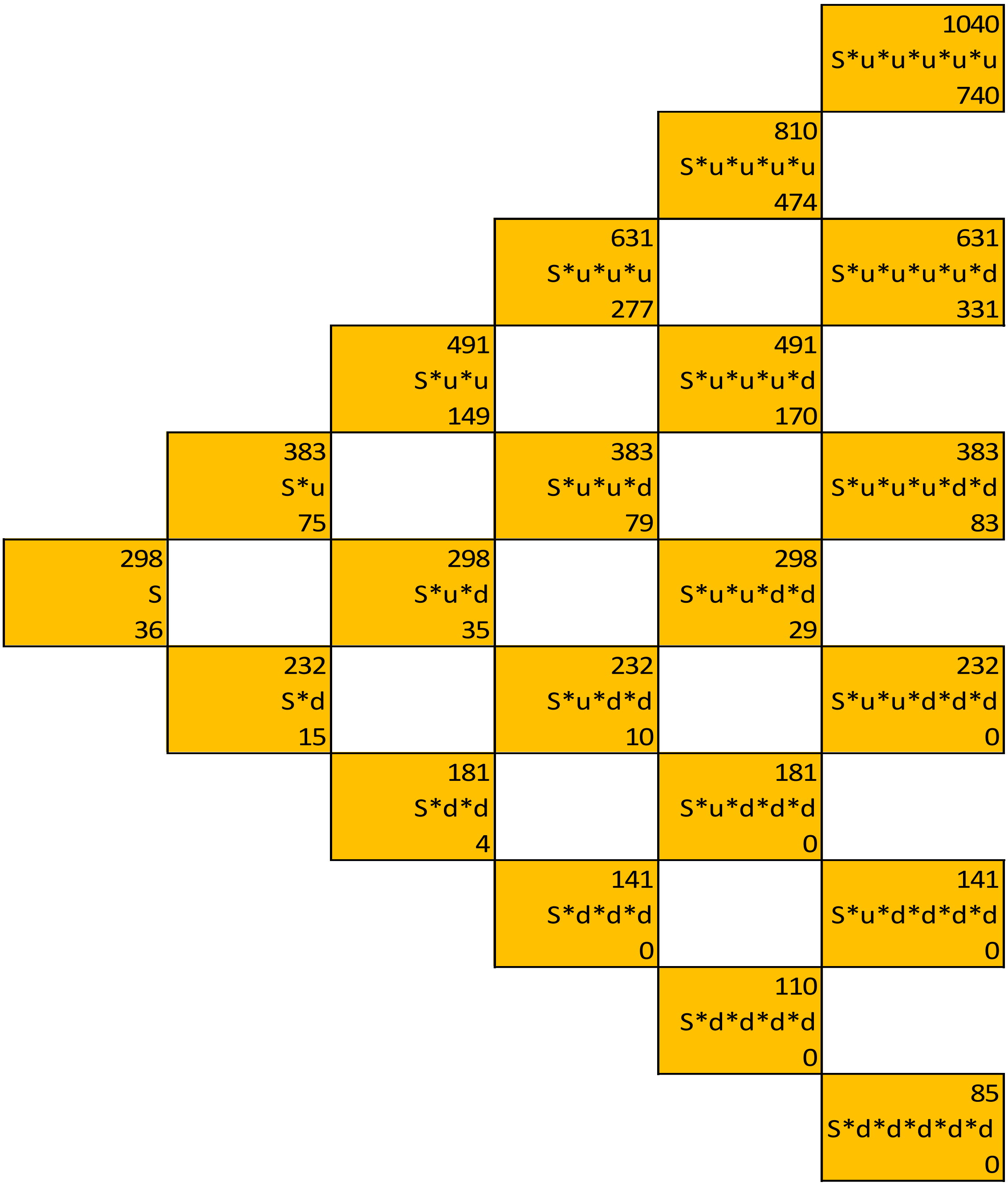

Fig. 5-7: The binomial lattice in case of leakage (all values $*10^6$).

5.2.4 Black and Scholes

The received option value of $ 36 MM is now compared to the analytical method of Black and Scholes.

The intermediate option parameters in this case are:

$$(85)\ d1 = \ln\left(\frac{S}{X} + (\text{risk free rate} + 0.5 * \text{volatility}^2) * \text{time}\right) * \frac{1}{\text{volatility} * \sqrt{\text{time}}}$$

$$(86)\ d2 = d1 - \text{volatility} * \sqrt{\text{time}}$$

According to (85) and (86)

$$d1 = \ln\left(\frac{298}{300} + (0.007 + 0.5 * 0.25^2) * 5\right) * \frac{1}{0.25 * \sqrt{5}} = 0.303$$

$$d2 = 0.303 - 0.25 * \sqrt{5} = -0.256$$

The standard normal distribution for d1 and d2 are 0.619 and 0.399 respectively.

As a consequence the option value according to Black and Scholes is

$$(87)\ C = N(d1) * S - N(d2) * X * \exp(-\text{risk free rate} * \text{time})$$

or

$$(88)\ C = 0.619 * 298 - 0.399 * 300 * \exp(-0.007 * 5) = 69 \text{ million}$$

The options value determined with Black and Scholes is $ 69 million compared to the $ 74 MM from the binomial lattice approach.

The difference between the analytical and the numerical binomial lattice approach is small and negligible. In contrast the NPV is with $ -2 million slightly negative and based on the NPV management will most likely not invest in the project. However the value of flexibility is reasonable high and by observing the main uncertainties the project might be in the money within a few years. Because of the annual leakage of 5 % the corresponding options value is only $ 36 million. Leakage is in financial options due to payouts (= dividends) and in case of an oil field it could be due to competitive losses.

5.3 OMV West Africa: Opportunity to Grow

Let's consider an oil and gas company that like in the last example acquired a license in West Africa and currently already produces hydrocarbons for some years. Because of a strategic change the operator of a nearby field would like to sell the license. The DCF valuation performed by OMV using WACC indicates a present value of $ 80 million. The estimated cost of the expansion will be only $ 120 million because OMV could use parts of their own already existing infrastructure and apply proven new production technologies in the acquired field so that the asset value could be doubled. This has to be decided within the following four years.

The annual volatility of the returns is assumed to be 20%. The continuous annual risk free rate is according to figure 2-3 for the option`s life of four years about 0.7 %. The value of the corresponding option to expand needs to be calculated using Black & Scholes as well as the so called binomial lattice method. The result will be analyzed and compared to the net present value.

The on the WACC based DCF yields $ 80 million. If the operation is expected to expand today the created value would be 2*80-80 equals 80 million. By subtracting the investment costs the net present value is $ -40 million.

By only considering the net present value which is significantly negative the nearby license won`t be acquired.

5.3.1 The Option to Expand

To accurately evaluate the chance to buy a nearby field the company decided to use the real options approach. Management is aware of the fact that especially in high-growth companies this option might heavily affect the final decision. Even if the NPV seems to be considerably low this opportunity could be valuable because the company could use the already existing infrastructure. To overcome the short-term outlook and improve the evaluation process the value of the option to expand will be calculated using the binomial approach.

According to the six step procedure, the first step involves framing of the application which was already done in the last paragraph. The second and the third step include the definition of the input parameters and the calculation of the intermediate option parameters.

The option parameters are:

- the underlying asset value of $ 80 million resulting in a net present value of $ -40 million for exercising the option immediately
- the time to expiration is 4 years
- the project volatility was estimated to be 20 %
- the risk free interest rate is according to Fig. 2-3 close to 0.7 %
- the investment costs for acquiring the nearby license are estimated to be $ 120 million
- the annual leakage rate is not considered in this example
- the time increment is set at 1 year which results in a valuation tree of four steps

The option parameters according to the already given equations representing the up and down movement as well as the risk neutral probability are:

$$\mathrm{u} = \exp\,(0.20 * \sqrt{1}) = 1.221$$

$$d = \frac{1}{1.284} = 0.819$$

$$p = \frac{1.007 - 0.819}{1.221 - 0.819} = 0.468$$

The following step incorporates the creation of the binomial tree with all together four steps, one for each year. Starting with the current asset value at time zero at the left side the corresponding asset value for the consecutive years will be calculated using the up and down factors, u and d. The top values in the tree represent the asset value at each year.

Now the options value needs to be estimated by backward induction. Therefore at each node the maximum value of continuing the option and keeping it open for another year or exercising the option now and paying the investment costs will be determined. Let`s start at the final node "$S*u^4$". If the company would invest in the expansion the asset value could be doubled if $ 120 million are invested. The result is 2*178 – 120 = 236 million. Because $ 236 million is greater than $ 178 million which is the options value in case of not exercising and continuing till next year, the company will invest and acquire the nearby field. At node "$S*u^2*d^2$" the asset value with no expansion is $ 80 million and with expansion 2*80-120 = 40 million. To maximize returns the company will not execute the option because expansion will result in less money. Therefore the options value at this node is $ 80 million instead of $ 40 million. The intermediate nodes are again the discounted weighted average of potential future option values using the determined risk neutral probability (Fig. 5-8).

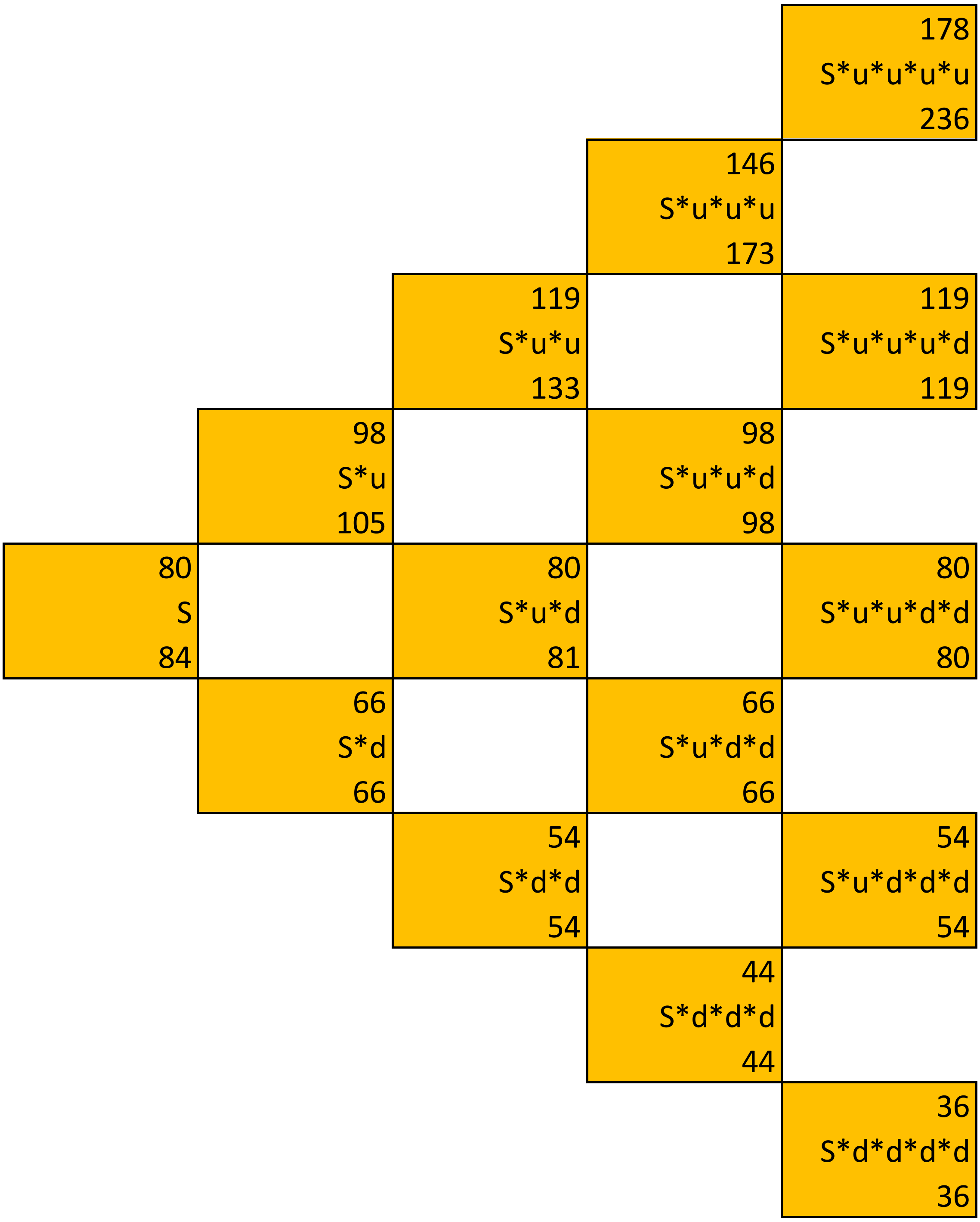

Fig. 5-8: The binomial lattice for an option to expand (all values $*10^6$).

The option to expand suggests a slightly higher value. By subtracting the \$ 80 million from the options value of \$ 84 million at time zero the present value would be \$ 4 million. By comparing this to the DCF and the corresponding negative value of \$ -40 million the additional value of flexibility is 4 – (-40) = \$ 44 million. Important to note is that the assumed input parameters are kept constant over the life time of the project.

If the expansion factor is instead of 2 only 1.5 the value of the option would decrease significantly. Instead of \$ 84 million the corrected value is then only \$ 80 million and the present value will be zero.

Calculating the options value will Black and Scholes yields an economic value of

$$\mathrm{C} = 0.2285 * 80 - 0.1264 * 120 * \mathrm{e}^{-0.007*4} = 3.5 \text{ million}$$

which is close to the \$ 4 million resulting from the binomial lattice approach. So Black and Scholes offers a quite accurate first estimate of the options value.

5.4 OMV West Africa: Sources of Uncertainty

After exploring the offshore oil field in West Africa OMV decides to look into building a new refinery. Because not only the oil prices are considered to be uncertain but also the production from the oil field due to technical issues two different and independent volatility factors should be used to evaluate the projects economics.

5.4.1 The Rainbow Option

These two indentified and completely separated uncertainty factors require a more advanced option called rainbow option. Only for this type of option it is possible to consider two main sources of uncertainty, e.g. technical and market. The technical uncertainty is solely based on experience. Even if the principle of the binomial lattice stays the same the two separated uncertainties can only be incorporated in a quadrinomial tree. This involves the definition of four up and down movements instead of just two and two separate risk neutral probabilities.

For the given example the input parameters are:

- refining the crude oil and selling the product on the local market in West Africa will result in a present value of \$ 80 million depending on the recoverable crude oil of the nearby field and the oil price
- the production volatility due to technical issues was estimated to be 20 % (corresponding up and down factors u1 and d1)
- whereas the oil price volatility depending on the local market is close to 30 % (corresponding up and down factors u2 and d2)
- plant construction is expected to cost \$ 250 million
- the time to expiration is 2 years
- the risk free interest rate is according to Fig. 2-3 close to 0.2 %
- the annual leakage rate is not considered in this example
- the time increment is set at 1 year which results in a valuation tree of two steps

Furthermore the option parameters are:

$$\mathrm{u1} = \exp\,(0.20 * \sqrt{1}) = 1.221$$

$$\mathrm{d1} = \frac{1}{1.284} = 0.819$$

$$p1 = \frac{1.002 - 0.819}{1.221 - 0.819} = 0.455$$

In contrast considering the market uncertainties the intermediate options parameters are:

$$u2 = \exp\,(0.30 * \sqrt{1}) = 1.350$$

$$d2 = \frac{1}{1.350} = 0.741$$

$$p2 = \frac{1.002 - 0.741}{1.350 - 0.741} = 0.429$$

The quadrinomial tree will be created and the asset values at each node be calculated according to the same already described principles.

The option value at intermediate nodes is again the discounted weighted average of potential future values but now considering four possible outcomes. For example for the node "$S*u^1*u^2$" this would be then:

$$[p1 * p2 * (S * u1 * u2 * u1 * u2) + p1 * (1 - p2) * (S * u1 * u2 * u1 * d2) + (1 - p1) * p2 * (S*u1*u2*d1*u2) + (1 - p1 * (1 - p2) * (S * u1 * u2 * d1 * d2)] * \exp(-\text{risk free rate} * \text{time increment}) = 21 \text{ million}$$

		358 S*u1*u2*u1*u2 108	197 S*u1*u2*u1*d2 0
	132 S*u1*u2 21	240 S*u1*u2*d1*u2 0	132 S*u1*u2*d1*d2 0
		108 S*u1*d2*u1*u2 0	59 S*u1*d2*u1*d2 0
	72 S*u1*d2 0	72 S*u1*d2*d1*u2 0	40 S*u1*d2*d1*d2 0
80 S 4		161 S*d1*u2*u1*u2 0	88 S*d1*u2*u1*d2 0
	88 S*d1*u2 0	108 S*d1*u2*d1*u2 0	59 S*d1*u2*d1*d2 0
		49 S*d1*d2*u1*u2 0	27 S*d1*d2*u1*d2 0
	49 S*d1*d2 0	33 S*d1*d2*d1*u2 0	18 S*d1*d2*d1*d2 0

Fig. 5-9: The quadrinomial lattice for a rainbow option with two separate sources of uncertainty (all values $*10^6$).

The net present value of the presented example is with 80 – 250 = -170 million highly negative. In contrast the calculated real options value is marginally positive. However, the value of flexibility that can only be considered in the real options approach is with 170 + 4 = 174 million remarkable. This is due to two different sources of uncertainty that in case of favorable evolution can provide this high additional value. In the real world this might still be unlikely but possible and management that constantly observes the market and tries to get new information or even changes the basic conditions could generate a positive economic value. In general the calculations of the quadrinomial tree are cumbersome and represent a so called non recombining tree. A Black and Scholes calculation of the option value is in case of different independent sources of uncertainty not possible.

6 Conclusion

Real options are an advanced tool that can incorporate not only the value of information but also the value of flexibility.

The value of flexibility is defined by the possible actions of the management on the one hand and the amount and number of independent sources of the projects uncertainty. The higher the uncertainty is the higher the possible risks and chances. In case of the discounted cash flow analysis and the corresponding net present value as one of the most prominent key performance indicators only the risks but not the chances can be considered. Therefore the NPV will always represent a lower economic limit. Nevertheless the NPV as well as other indicators like the internal rate of return or the profitability index need to be included in the evaluation process. Besides the real options approach cannot replace either the NPV nor Monte Carlo methods or the decision tree analysis. It is rather an advancement of the DCF and can in some models be seen as an "addon" to the classical discounted cash flow analysis.

However, to gain a better insight and be able to improve decision making the real options approach has its value. Originally this technique was developed to estimate the value of an option on the financial market. This mathematic principles mainly influenced by the Nobel prize winners Black and Scholes was further adjusted to be able to use for real assets. Although these classical models like the one from Black and Scholes which represent an analytical solution of the partial differential equation describing the change of the value of an option over time on the market can be used for the evaluation of real assets some crucial limitations need to be considered (e.g. an efficient and arbitrage free market). Regardless of these restrictions the model offers an easy and fast but mathematically difficult method to get a first impression of the options value.

Advanced real option techniques provide a more detailed view and in case of the binomial lattice approach can even illustrate the optimal decision path. By calculating the pros and cons at each node in the tree this technique is able to determine by backward induction the today's option value.

The in this master thesis described process is a simple six step procedure. After framing the application and definition of inputs as well as intermediate option parameters the tree can be built and analyzed. Even if the graphical representation is usually called a tree the calculated probabilities are rather weighting factors than real objective probabilities like in case of decision trees.

Many projects might have two independent sources of uncertainty – perhaps one driven by the market itself and one because of technical or HR related project issues. Also for this situation a quite cumbersome quadrinomial lattice approach was developed. The corresponding type of the option is called rainbow option.

The above described approaches were used to evaluate different stages of an E&P project in West Africa. The first example covered the exploration whereas the second and third were related to the development and production. All examples show that Black and Scholes are a valid first estimate. The difference between the classical and the more accurate lattice approach is marginal. However, Black and Scholes do not offer the same insight and the different time steps cannot be graphically demonstrated. Nevertheless Black and Scholes or any other approximation of the PDE provides a fast and reasonable solution.

On the other hand the binomial as well as the quadrinomial lattice technique can visually represent the best possible path and the highest outcome. This is a huge advantage over any black box model.

The net present value was in all examples negative whereas the options value of all three considered types is even in the highly negative case still slightly positive. Therefore in the author's opinion the real option approach definitely defines an upper limit compared to the NPV.

In general the real options approach is a valid tool that is technically feasible. However, management often still needs to be persuaded to accept and support it. Especially in today's fast changing risky environment this advanced method offers a great technique to complete the valuation process by adding a new key indicator.

Even if the calculations can sometimes be cumbersome like in the quadrinomial lattice approach the mathematics from a practitioners point of view is rather simple. Real options can add a significant competitive advantage and should be integrated in project evaluation as well as project selection. Moreover real options are no replacement of traditional methods but this technique can fill the gaps and finally estimate the value of flexibility. Black box models should be avoided and only used for verification. The best way to communicate results is the presented transparent lattice approach. This should allow high acceptance of management and assure the needed support.

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