Imbibition capillary pressure curve modelling for two-phase flow in mixed-wet reservoirs

Author: Julia Maria Inés Bruchbacher, BSc.

Supervisors: Univ.-Prof. Dipl.-Geol. PhD. Stephan K. Matthäi
Prof. Svein M. Skjæveland

MONTAN
UNIVERSITÄT
WWW.UNILEOBEN.AC.AT

University of
Stavanger

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Abstract

Until the early 2000s, the majority of reservoirs worldwide were considered to be either water-wet or oil-wet and capillary pressure correlations were developed subsequently. Recently, it was shown that most reservoirs are mixed-wet (Anderson 1986, Delshad et al. 2003, Lenhard and Oostrom 1998) and methods and techniques available to evaluate capillary pressure curves in such a media are limited.

To advance on this topic, the current thesis deals with the modelling of capillary pressure curves in two-phase, mixed-wet reservoirs and proposes a way to evaluate capillary pressure experiments.

The proposed method aims to obtain both positive and negative imbibition capillary pressure curves using saturation profiles gained from a centrifuge experiment. The saturation data gathered from an artificially created centrifuge experiment is used to determine the following parameters: residual oil saturation, irreducible water saturation, pore size distribution indices as well as the capillary entry pressure for the non-wetting and wetting phases. This process is performed using a combination of a correlation and a centrifuge experiment. The correlation is modelled and implemented in Maple and support with a tool established in Visual Basic. The centrifuge experiment is simulated in Maple and imbibition capillary pressure hysteresis curves are produced using the concept by Skjæveland et al. (1998), which is the preferred correlation for mixed-wet reservoirs.

Artificially created saturation data is used in the model as a first trial, to evaluate if the procedure can work and the presented model leads to acceptably results. The performed curve fitting achieves high accuracy to match the model with generated test data used to create the saturation profile.

Follow ups for field application of the developed Maple tool are proposed and an outlook for the difficulties facing three phase flow is given.

Kurzfassung

Bis zum Beginn des 21 Jahrhunderts, wurden weltweit die meisten Lagerstätten als wasser- oder öl-benetzbar eingestuft und für diese wurden Kapillardruck Korrelationen entwickelt. In letzter Zeit wurde nachgewiesen, dass die meisten Lagerstätten jedoch misch-benetzt sind (Anderson 1986, Delshad et al. 2003, Lenhard and Oostrom 1998) und die verfügbaren Methoden und Techniken sind in diesen Lagerstätten begrenzt um Kapillardruckkurven auszuwerten.

Um in diesem Themagebiet Fortschritte zu machen, handelt diese Arbeit vom Modellieren von Kapillardruckkurven im Zweiphasenfluss in misch-benetzten Lagerstätten und stellt eine Methode vor, um Kapillardruck Experimente zu evaluieren.

Das Ziel der vorgestellten Methode ist es sowohl positive als auch negative Imbibition-Kapillardruckkurven zu erhalten, unter Verwendung von Sättigungsprofilen, welche von einem Zentrifugenexperiment bezogen werden. Die Sättigungsdaten werden von einem künstlich erstellten Zentrifugenexperiment erhalten, um die folgenden Parameter zu bestimmen: nicht reduzierbare Ölsättigung, irreduzible Wassersättigung, Porengrößen-Index als auch den Kapillareingangsdruck für beide Phasen. Dieser Vorgang wird durch die Kombination aus einer Korrelation und einem Zentrifugenexperiment durchgeführt. Die Korrelation wird in Maple modelliert und implementiert sowie durch ein weiteres Tool in MS Excel Visual Basic unterstützt. Das Zentrifugenexperiment wird in Maple simuliert und die Imbibition-Kapillardruck-Hysteresis-Kurven werden mit der Korrelation von Skjæveland et al. (1998), die bevorzugte Gleichung für misch-benetzte Lagerstätten, erstellt.

Künstlich erstellte Sättigungsdaten werden in dem Model als erster Versuch verwendet, um zu testen, ob die vorgestellte Methode funktioniert und zu akzeptablen Ergebnissen führt. Durch einen Vergleich der Sättigungskurven, der künstlich erstellten Testdaten und denen des Modells, kann eine hohe Übereinstimmung festgestellt werden.

Es werden weitere Schritte zur Verwirklichung des erstellten Tools präsentiert, sowie ein Ausblick auf die auftretenden Schwierigkeiten, die bei Dreiphasenströmung in einer Lagestätte überwundern werden müssen.

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Nomenclature

Symbol	Description	Unit
a	pore size distribution index	[-]
b	fitting parameter	[-]
C	Land's trapping constant	[-]
h	height	[m]
p	pressure	[Pa]
r	radius	[m]
S_1	saturation crossover point (P_{ci})	[-]
S_2	saturation crossover point (P_{cd})	[-]
S	saturation	[-]
ρ	density	[kg/m³]
ω	speed of rotation	[RPM]
$[k]$	scanning loop reversal No. k	[-]

Subscripts	Description
c	capillary or connate
d	drainage
cd	capillary entry pressure
g	gas
i	Initial or imbibition
o	oil or oil-wet
r	residual or irreducible
w	water or water-wet
0	zero point (p_c=0)

Superscripts	Description
dra	drainage
imb	imbibition
$*$	effective

Abbreviations

Acronyms	Description
FWL	Free Water Level
G°	Gibbs Free Energy
MRI	Magnetic Resonance Imaging
OWC	Oil Water Contact
PID	Proportional, Integral and Derivative Control System
PWC	Pumping While Centrifuging
RPM	Revolutions Per Minute
SCAL	Special Core Analysis
TZ	Transition Zone

1. Introduction

Capillary pressure is an important factor behind multi-phase flow behavior (Green et al. 2008) and capillary pressure curves are input to models predicting flow in hydrocarbon reservoirs. Multi-phase flow predictions with inaccurate capillary pressure input can lead to incorrect prediction of watercut, especially in heterogeneous reservoirs. Inefficient depletion plans with large scale investments in facilities that cannot process the produced fluids may result (Masalmeh, Abu Shiekah and Jing 2007).

Capillary pressure also determines oil saturation in the transition zone and therefore the oil in place. Transition zones are often assumed to be mixed-wet. Furthermore the transition zone can contain a large amount of the initial oil in place (Carnegie 2006, Masalmeh et al. 2007) and can vary between just a few meters up to a hundred meters depending on reservoir characteristics (Masalmeh et al. 2007). Any contact movements can be crucial for production and incorrect prediction can lead to undesired coning effects. Estimates of recovery efficiency can therefore only be made if capillary pressure effects are understood. An error in the transition zone capillary pressure can therefore lead to large-scale errors in STOIIP estimates. The height of the transition zone in a reservoir is determined by the earth's gravitational field and may be compressed to the cm-scale using a centrifuge. The height of the transition zone is derived from a capillary pressure versus saturation profile. Therefore for modelling transition zones properly capillary pressure curves models are very important. Especially imbibition capillary pressure scanning curves play an important role for the transition zone as crossflow between the high and low permeability layers is improved. This leads to a better recovery and protracts the water breakthrough.

However not only transition zones are considered to be mixed-wet nowadays almost all reservoirs are considered to be mixed-wet (Delshad et al. 2003, Lenhard and Oostrom 1998). Until 2000, most reservoirs were considered to be water- or oil-wet and therefore most present techniques for capillary pressure interpretation have been developed for water- or oil-wet reservoirs. For many years it was assumed that sandstone reservoirs are strongly water-wet and carbonate reservoirs are oil-wet. However with the improvement of lab methods and coring techniques it was discovered that for most rocks both phases are wetting. The previously assumed wetting phase, i.e. water for sandstone and oil for carbonate, is trapped in the large pore but is still partially adhering to the rock surface, making it mixed-wet (Radke et al. 1992). Nowadays it is essential to further improve the research that has been done for mixed-wet reservoirs.

An additional limitation is that the majority of capillary pressure interpretation techniques have been developed for two phases while most reservoirs contain three phases in reality. To describe the flow in three-phase reservoirs where capillary pressure differences exist between oil and gas and oil and water, the determination and interpretation of capillary pressure curves is subsequently more complex and requires combination of two capillary pressures. To find a correlation for three-phase flow, two-phase capillary effects have to be modelled first in a right way and the available methods reviewed.

Four main types of lab methods can be used to obtain capillary pressure - saturation curves: centrifuge, porous plate, membrane and mercury injection. In this thesis, centrifuge experiments are discussed in detail, forming the basis of the work. Porous plate experiments are usually more precise but the measurement of a capillary pressure point takes weeks to months. As improvement for the porous plate method the micro pore membrane technique can be used (Hammervold et al. 1998). In contrast, mercury injection is quick and high capillary pressure values can be obtained. The main disadvantages are that the core is destroyed and mercury is a non-representative reservoir fluid. Centrifuge methods use reservoir fluids and are not as time consuming as porous plate methods (Green et al. 2008). The problem with centrifuge experiments is that only negative imbibition and drainage curves can be obtained. The positive capillary pressure region is cumbersome to investigate experimentally due to hysteresis effects and is often calculated using correlations.

There are multiple techniques available in the literature on how to use the experimentally obtained capillary pressure data and interpret primary drainage curves. Primary drainage capillary pressure curves are easier to interpret. Dealing with imbibition capillary pressure curves hysteresis effects are more essential. Experimental methods available (e.g. Fleury et al. 1999) often circumvent this hysteresis effect for imbibition by establishing uniform residual saturation of the core sample after the primary drainage or simply neglect it (e.g. Spinler and Baldwin 1997).

Besides experimental methods, correlations can be used to describe capillary pressure curves. As there are many correlations for capillary pressure curves in water-wet reservoirs (Skjæveland et al. 1998), the recent focus of research are correlations for mixed-wet reservoirs. Skjæveland et al. (1998) developed a widely used correlation incorporating hysteresis effects (Eigestand & Larsen 2000, Bech, et al. 2005, Pirker et al. 2007, Kralik et al. 2010, Abeysinghe et al. 2012a,b, Hashmet et al. 2012, El- Amin et al. 2013).

In this thesis I describe the modelling of capillary pressure curves in mixed-wet, two-phase reservoirs and propose a new way to evaluate capillary pressure experiments. I demonstrate a method to obtain both positive and negative imbibition capillary pressures curves using results from artificially created centrifuge experiments. The main challenge is to find a way of including the hysteresis effect in the evaluation and interpretation for imbibition capillary pressure curves. The idea explored here is based on using the capillary pressure correlation for mix-wet reservoirs by Skjæveland et al. (1998) as well on the two mentioned experimental techniques. These two techniques were the main motivation for my thesis. With the support of my tool, I want to discuss why the presented techniques are not accurate enough and that a new method would be desirable. Of course some of their ideas were very useful for me and therefore the two methods are explained in detail. With the help of their ideas behind the centrifuge experiments and the correlation, I developed the idea to use both methods to derive imbibition capillary pressure curves incorporating the hysteresis effect.

The importance of further improvement in this area was highlighted, especially for mixed-wet reservoirs too little prospects are present and therefore a new technique is evaluated. The thesis contains an extensive literature review where general definitions of capillary pressure curves and residual saturations are presented. Furthermore the correlation by Skjæveland et al. (1998) is explained as it is used to interpret centrifuge experiments to determine capillary pressure curves. Then two different centrifuge techniques are introduced which claim to establish capillary pressure curves and will be discussed later. The mentioned correlation constitutes the basis of modelling the capillary pressure curves. The created program establishes drainage and imbibition capillary pressure bounding curves as well as scanning curves including hysteresis effects. Subsequently a model to simulate a centrifuge experiment calculating capillary pressure and saturation, accounting for centrifugal forces, is created. The obtained saturation profiles from the program are compared through curve fitting with saturation profiles obtained from an artificially centrifuge experiment. To find the minimum deviation between the artificially created test data and the saturation obtained with the Maple program, residual saturations, pore size distribution indices and capillary entry pressure for wetting and non-wetting phases, are adjusted with the Ms Excel solver. With the titled parameters it is possible to create imbibition capillary pressure curves which incorporate hysteresis.

In the outlook an overview of existing correlations for three phases in mixed-wet reservoirs and their limitations are presented.

2. Background information

To describe the flow of three phases, oil, gas and water reservoir parameters have to be considered and they have a large influence on capillary pressures. The capillary pressures between oil and water and oil and gas need to be determined and then combined to an integrated system. To find a correlation that fits experiments and can describe three-phase flow, it is important to understand which factors affect the capillary pressure curve. Two-phase capillary pressure curves are defined through residual oil saturation, irreducible water saturation, oil and water saturations, pore geometry, capillary entry pressure, permeability and porosity. Therefore it is important to know the interaction of these factors first in two-phase flow that it can be extended to three-phase flow.

To understand the created tool, first the basics of all cooperating parts need to be known. Therefore I am going to start with an explanation of basic knowledge that is required. Furthermore this part of the thesis should help to understand the necessity of deriving capillary pressure curves.

2.1 Definition of drainage and imbibition

An idealized capillary pressure curve for a mixed-wet reservoir is shown in Figure 1.

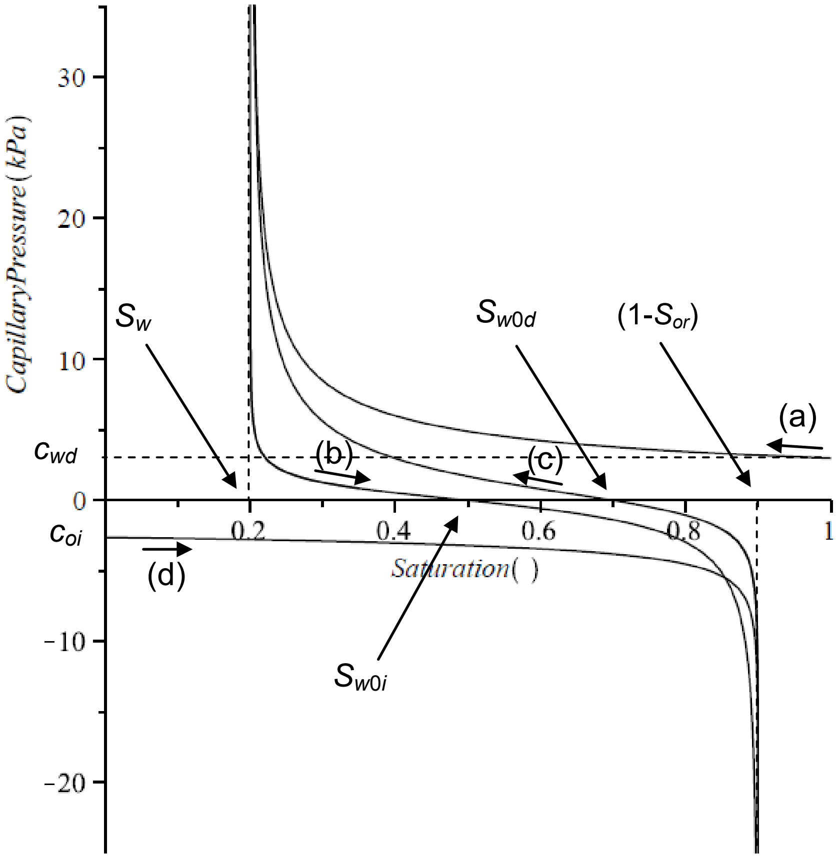

Figure 1 Schematic of bounding curves for mixed-wet reservoir. (a) primary drainage, (b) (secondary) imbibition, (c) secondary drainage and (d) primary imbibition.

Drainage is used to describe a process where the wetting phase saturation is decreasing. It is called spontaneous drainage (invasion) if the capillary pressure is negative and it is called forced drainage when it is positive. Primary drainage is when the drainage process starts at 100 % wetting phase saturation.

Imbibition is used to describe a process where the wetting phase saturation is increasing. It is called spontaneous imbibition if the capillary pressure is positive and forced (injection) if it is negative. Primary imbibition describes the imbibition process starting at 100 % non-wetting phase saturation.

Bounding loop is the outer loop, starting at the lowest irreducible water saturation and ending in the lowest residual oil saturation.

Scanning loops are all loops inside the bounding loop.

2.2 Residual saturations

To establish a correlation for two-phase capillary pressures, it is necessary to know the residual saturations of all phases. Incorrect residuals lead to wrong results in the capillary pressure models. In the literature there are different ways proposed to obtain the irreducible saturations.

The residual saturation, the fraction of the phase which cannot be recovered, depends on the pore structure. Therefore depending on the rock and fluid system, different techniques are in use to determine residual saturation. An often used correlation to find residual saturations is Land's Correlation (Land 1967, 1971). It serves a basis function adapted in different ways to fit the data. Land assumed that during the imbibition process the non-wetting phase consists out of two different parts. One part is considered to be the residual saturation and therefore does not account to flow and the other one is the mobile section, which is used as the actually non-wetting saturation. The mobile non-wetting phase saturation is obtained from the residual non-wetting phase saturation after imbibition starting from initial non-wetting phase saturation in draining direction. The residual gas saturation is received from laboratory measurements. The following relationship between initial and residual gas saturation is assumed:

$$\frac{1}{S_{gr}^{*}} - \frac{1}{S_{gi}^{*}} = C\,. \tag{1}$$

S^{*}_{gr} … effective residual gas saturation [-]
S^{*}_{gi} … effective initial gas saturation [-]
C … trapping constant [-]

The trapping constant defines the trapping capacity of a rock. The effective saturations refer to the pore volume excluding the occupied pore volume of the irreducible wetting phase. The correlation works well for water-wet sandstones. However there are also studies available that question the validity of Lands relationship, especially in unconsolidated sand packs. Others claim that the Aissiaouri correlation works best in this environment (Iglauer 2009). Masalmeh (2007) showed that Land's correlation in mixed-wet/oil-wet reservoirs leads to incorrect results. Some research projects state that Land's correlation works, if special core analysis (SCAL) data is available to adjust it. It is obvious that for every geologic facies another correlation is needed. Skjaeveland et al. (1998) adapted Land's correlation to mixed-wet reservoirs (Chapter 2.4). However not only correlations can be used to determine residual saturations, different methods propose how the residuals can be analyzed with the help of core/sand pack experiments (Pentland 2010).

Three-phase measurements of residual saturations are more difficult than for two-phase reservoirs. Al-Mansoori (2009) observed that in three-phase flow in unconsolidated sand packs the residual gas saturation can be higher than the one in two-phase systems, where only water is present. That differs from measurements in consolidated media where the irreducible gas saturation is similar or lower than in a two-phase system. Also the amount of the residual oil is not effective to the initial oil saturation and more oil is trapped than in a comparable two-phase experiment. This results from the piston-like displacement in siliciclastic two-phase water-wet reservoirs, which leads to relatively little trapping and therefore to lower residual saturations (Al-Mansoori 2009). In consolidated media snap-off can occur as the throats are much smaller than the pores. In water-wet media gas is a non-wetting phase and therefore gets trapped by snap-off. The degree of trapping is independent of the initial oil saturation.

The determination of residual saturations is a prerequisite to establish reasonable capillary pressure curves (Al-Mansoori 2009, Iglauer 2009, Pentland 2010).

2.3 Transition zones

The transition zone is a reservoir interval from the oil-water contact (OWC) up to the level where the irreducible water saturation is reached. In Figure 2, a typical transition zone of a homogenous reservoir is illustrated. The transition zone is controlled by the balance of capillary and buoyancy forces during the primary drainage process. The so-called capillary entry pressure or threshold pressure (P_{cd}) is the pressure in the largest pores, which has to be overcome that the oil can start to enter the pore. The height of the transition zone and its saturation distribution is controlled by the following factors: range and distribution of pore sizes, interfacial force and the density differences between the fluids.

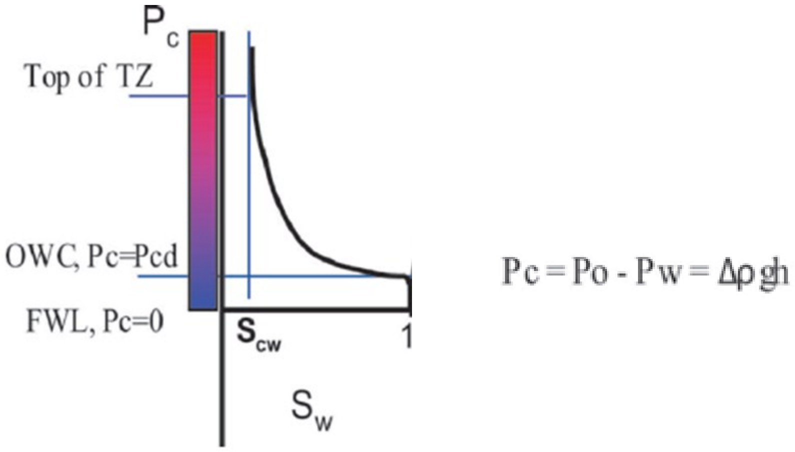

Figure 2 Schematic of a transition zone (Figure 1, Masalmeh et al. 2007). The transition zone for a drainage capillary pressure curve for a homogenous reservoir with both the water and the oil phases are mobile. TZ is the acronym for transition zone, P_o for the oil pressure, P_w for the water pressure S_{cw} for the connate water saturation and *h* for the height of the transition zone.

The amount of producible oil in the transition zone is contingent on initial oil saturation distribution as a function of depth, the relative permeability and capillary pressure characteristics. Capillary pressure curves including hysteresis have a significant influence on field performance predictions especially for heterogeneous reservoirs with transition zones.

The wettability may vary as a function of height above the FWL and the initial water saturation and often becomes mixed-wet. Especially for mixed- /oil-wet heterogeneous reservoirs the probably most important parameter influencing water flooding is imbibition capillary pressure. It can help to prevent cross-flow between different layers which leads to a poor sweep efficiency. The imbibition capillary pressure scanning curves plays an important role as well as it improves the cross-flow between high and low permeability layers. This yield to a later water breakthrough and a better recovery compared to a case where only imbibition bounding curves are used. This shows that recovery is strongly dependent on capillary pressure models as well as on the details of the geological heterogeneity. Therefore it is crucial to model capillary pressure curves in mixed-wet reservoirs (Masalmeh et al. 2007).

2.4 Two-phase capillary pressure correlations

The correlation developed by Skjaeveland et al. published in 1998 is one of the most widely used in the industry (Eigestand & Larsen 2000, Bech et al. 2005, Pirker et al. 2007, Kralik et al. 2010, Abeysinghe et al. 2012a,b, Hashmet et al. 2012, El- Amin et al. 2013). It is the basic concept used to model centrifuge experiments in this thesis, to obtain capillary pressure curves including hysteresis, saturation profiles and to solve for residual oil saturation, irreducible water saturation, capillary entry pressure as well as the pore size distribution index.

Skjæveland et al. adapted the simple power-law presented by Brooks and Corey (1964 and 1967) as basic equation for completely water-wet and oil-wet reservoirs. Thus for calculating the capillary pressure in a water-wet system,

$$P_c = \frac{c_w}{\left(\frac{S_w - S_{wr}}{1 - S_{wr}}\right)^{a_w}}, \tag{2}$$

and for an oil-wet system,

$$P_c = \frac{c_o}{\left(\frac{S_o - S_{or}}{1 - S_{or}}\right)^{a_0}}. \tag{3}$$

a … pore size distribution [-]	S_{or} … residual oil saturation [-]
c … entry pressure [Pa]	S_w … Water saturation [-]
P_c … capillary pressure [Pa]	S_{wr} … irreducible water saturation [-]
S_o … oil saturation [-]	

The basic idea is to sum the terms for water and oil branch up to a symmetrical form describing the capillary pressure of mixed-wet reservoir rocks. The subsequent equation is used for modelling the imbibition curve in a mixed-wet reservoir:

$$P_{ci}=\frac{c_{wi}}{\left(\frac{S_w-S_{wr}}{1-S_{wr}}\right)^{a_{wi}}}+\frac{c_{oi}}{\left(\frac{S_o-S_{or}}{1-S_{or}}\right)^{a_{oi}}}\,. \tag{4}$$

a_{oi} … imbibition pore size distribution for the oil phase [-]
a_{wi} … imbibition pore size distribution for the water phase [-]
c_{oi} … imbibition entry pressure for the oil phase [Pa]
c_{wi} … imbibition entry pressure for the water phase [Pa]
P_{ci} … imbibition capillary pressure [Pa]

To model the drainage curve the index “*i*” is substituted by “*d*” and the saturation interval *S* going from S_{or} to S_{wr}. For simplicity it is assumed that the two variable sets a_w, a_o, c_w, c_o, one set for drainage and one for imbibition, are constant.

To model the hysteresis loop a modification of Land’s correlation is used to calculate the residual saturations:

$$\frac{1}{S_{or}[1]}-\frac{1}{S_o[1]}=C\,. \tag{5}$$

Where *C* is the Land’s trapping constant and $S_o[1]$ the start- and $S_{or}[1]$ the end-point saturations of the imbibition process. The following assumptions are used to establish the capillary pressure hysteresis loop:

- The first saturation reversal (direction change) starts from the primary drainage curve before the residual water saturation is reached. It is an imbibition scanning curve which ends at the residual oil saturation.
- A bounding imbibition curve starts at residual water saturation on the primary drainage curve and scans to the residual oil saturation.
- A closed hysteresis loop is defined when drainage and imbibition curve build a closed loop. For example when a second reversal from the residual oil saturation scans back to the residual water saturation.
- All scanning curves that have their origin on the bounding imbibition curve scan back to the residual water saturation and vice versa all the reversals on the bounding imbibition curve come back to the residual oil saturation.
- A scanning curve which starts from $S_w[k]$ will scan back to $S_w[k-1]$ and form a closed scanning loop. This loop can only be discontinued if a new reversal occurs before $S_w[k-1]$ is reached. If no new reversal occurs the curve scans back to $S_w[k-2]$.

- The shapes of the bounding and scanning curves are similar.

The first process in a two-phase flow reservoir is often the primary drainage. Therefore the first saturation reversal will form an imbibition curve with the origin on the primary drainage curve. The reversal saturation is a point where both the imbibition and the drainage curve meet and can be determined as follows,

$$p_{cd}[0](S_w[1]) = p_{ci}[1](S_w[1])\,. \tag{6}$$

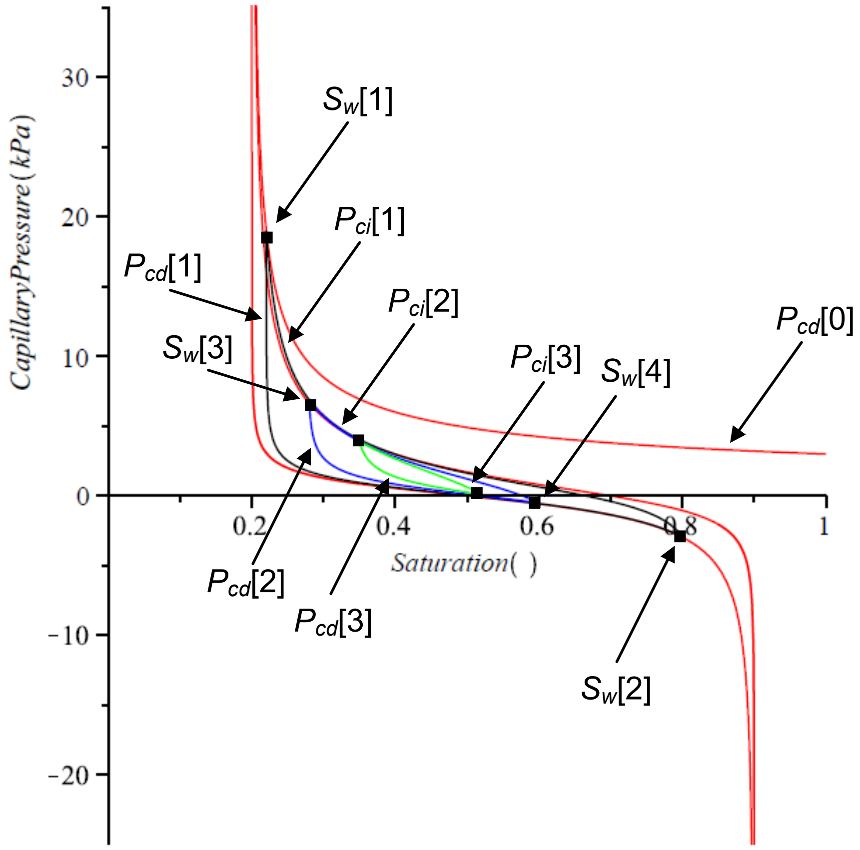

Figure 3 Drainage and imbibition capillary pressure curves. The saturation reversal points and scanning curves are illustrated. The program is created with Maple as discussed in Chapter 3.1. The black squares indicate the saturation reversal points named S_w[k]. P_{cd}[k] and P_{ci}[k] indicate the imbibition and imbibition capillary pressure curves.

Where $P_{cd}[0]$ is the primary drainage curve and $P_{ci}[1]$ the imbibition curve. Figure 3 shows the first reversal for the imbibition curve which has its origin on the primary drainage curve at the so-called reversal point $S_w[1]$ and ends in the asymptote $S_{or}[1]$. Therefore the second reversal starts at a point on the first imbibition curve before reaching the residual saturation of the first reversal or at the residual oil saturation. Then the drainage curve scans back to the first reversal point to form a closed loop and is evaluated with the following equation:

$$p_{cd}[2](S_w[1]) = p_{ci}[0](S_w[1])\,. \tag{7}$$

The reversal drainage scanning curve is created. This leads to a closed loop, as the imbibition curve from the first reversal and the drainage curve from the second reversal, are equal at the two reversal points (shown in Figure 3).
In general terms, the following two equations are used:

$$p_{ci}[1](S_w[1]) = p_{cd}[2](S_w[1])\,, \tag{8}$$

$$p_{ci}[1](S_w[2]) = p_{cd}[2](S_w[2])\,. \tag{9}$$

With these two equations the asymptotes $S_{wr}[2]$ and $S_{or}[2]$ for the second drainage curve are defined. The two equations are solved by estimating a value for $S_{wr}[2]$, as a first attempt the value of $S_{wr}[1]$ is used and then $S_{or}[2]$ can be calculated from Equation 8. Then Equation 9 is used to get a new value for $S_{wr}[2]$, the new value is inserted in the Equation 8. This iterative process continues until the values for $S_{wr}[2]$ and $S_{or}[2]$ converge. In Figure 3 the third reversal is reached when the process follows the secondary drainage bounding curve until a third reversal occurs at $S_w[3]$. The process continues on the third imbibition bounding curve to the water saturation point of the second reversal. Before this point is reached, a fourth reversal occurs at $S_w[4]$. The process continuous until the last reversal $S_w[k]$ occurs, then the process scans back on the drainage curve of the last reversal k to the point $S_w[k\text{-}1]$ and continues on the following drainage curve ($p_{cd}[k\text{-}2]$) this goes on until the secondary drainage bounding curve is reached.

The correlation by Skjæveland et al. (1998) was preferred over other available correlations (Delshad et al. 2003, Lenhard and Oostrom 1998, Lomeland and Ebeltoft 2008) as it is rational and not fully empirical. In the correlation, the wetting branch and the non-wetting branch are summed up which can result in an either-or solution or a symmetrical solution and therefore different fractions of wettability are considered. If the reservoir is more oil-wet, the oil branch has a bigger influence than the water branch and is displayed through the shape of the curve. Respectively for a more water-wet reservoir it is the other way around.

Further investigations of the correlation by Skjæveland et al. (1998) were performed and a modified correlation by Masalmeh et al. (2007) will be discussed.

2.4.1 Improved representation of scanning curves

To model capillary transition zones in another way, Masalmeh et al. (2007) modified the correlation by Skjaeveland et al. (1998). A third term is introduced which should account for the different shapes of scanning curves:

$$P_{cd} = \frac{c_{wd}}{\left(\frac{S_w - S_{wc}}{1 - S_{wc}}\right)^{a_{wd}}} + \frac{c_{od}}{\left(\frac{1 - S_w - S_{or}}{1 - S_{or}}\right)^{a_{od}}} + b_d * \left(S_{wcutoff}^{dra} - S_w\right). \tag{10}$$

$S_{wcutoff}^{dra}$ … Cutoff water saturation for drainage [-]

b_d … fitting parameter [-]

Equation 10 describes the bounding drainage capillary pressure. Corresponding to describe the bounding imbibition curve the subscript "*d*" is changed to "*i*" and the superscript "*dra*" to "*imb*". In the extension b_d/b_i is zero for water saturation higher than $S^{dra}_{w\_cutoff}$ / lower than $S^{imb}_{w\_cutoff.}$ The fitting parameter b is obtained from core data. The third term is used as the original model was not able to fit the experimental data, especially where the pore-size distribution is non-uniform (1/*a* describes the pore size distribution) and for measured imbibition capillary pressure curves. For calculating the imbibition curves scanning curves the following equation was used:

$$P_{ci} = \frac{c_{wi}(S_{oi})}{\left(\frac{S_w - S_{wi}}{1 - S_{wi}}\right)^{a_{wi}(S_{oi})}} + \frac{c_{oi}(S_{oi})}{\left[\frac{1 - S_w - S_{or}(S_{oi})}{1 - S_{or}(S_{oi})}\right]^{a_{oi}(S_{oi})}} + b_i(S_{oi}) * \left(S_{wcutoff}^{imb} - Sw\right). \tag{11}$$

The fitting parameters are determined as followed:

$$\begin{aligned} c_{wi}(S_{oi}) &= c_{wi} + c_{wi1} * (1 - S_{oi} - S_{wc})\,, \\ a_{wi}(S_{oi}) &= a_{wi} + a_{wi1} * (1 - S_{oi} - S_{wc}), \\ c_{oi}(S_{oi}) &= c_{oi} + c_{oi1} * (1 - S_{oi} - S_{wc})\,, \\ a_{oi}(S_{oi}) &= a_{oi} + a_{oi1} * (1 - S_{oi} - S_{wc})\,, \\ b_i(S_{oi}) &= b_i + b_{i1} * (1 - S_{oi} - S_{wc})\,. \end{aligned} \tag{12}$$

With the presented equations it is possible to calculate capillary scanning curves that fit the experimental data. Masalmeh et al. extension was not developed for general use, for the specific data set different fitting parameters are needed.

2.5 The centrifuge method

It was already mentioned that four experimental methods can be used to obtain capillary pressure curves (centrifuge, porous plate, a micro pore membrane and mercury injection). In this thesis only centrifuge experiments are used. The advantages of the centrifuge method compared to others are usage of representative reservoir fluids and shorter duration (Green et al. 2008). The disadvantage is that only negative imbibition and drainage curves can be obtained. The positive capillary pressure region is cumbersome to obtain experimentally due to hysteresis effects. The schematic of a centrifuge is shown in Figure 4.

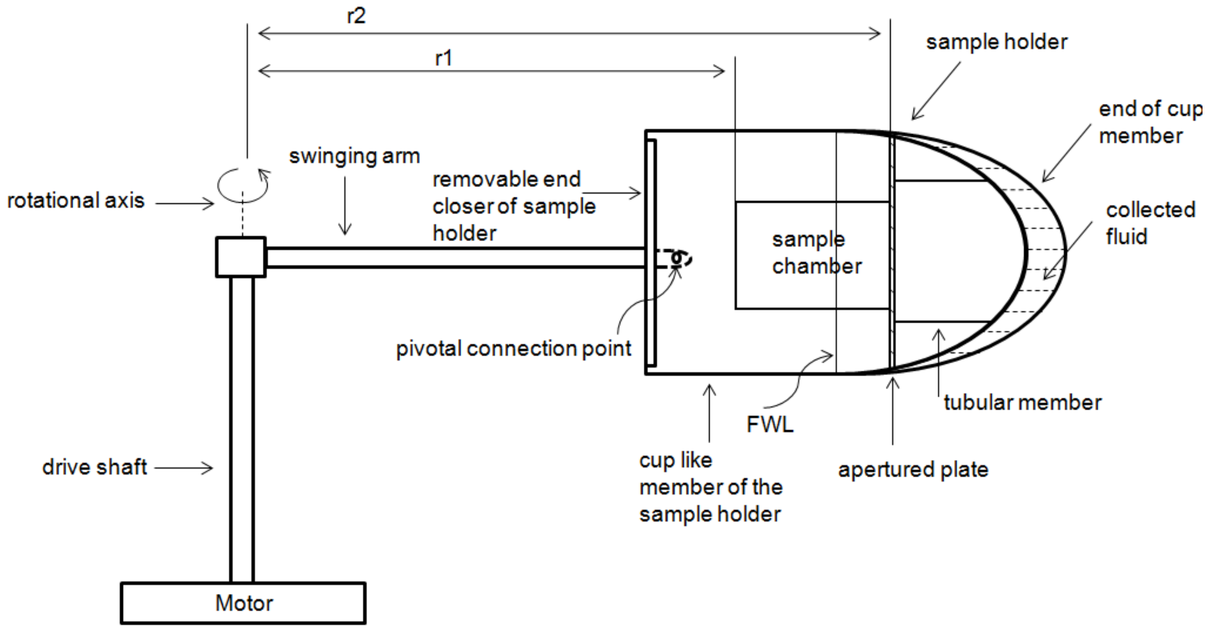

Figure 4 Schematic of a centrifuge. A schematic of a typical centrifuge is illustrated. The sample is placed in a huge bulk volume (sample holder). The radii (r_1 and r_2) used for calculating the capillary pressure are indicated in the sketch.

In a centrifuge experiment a small uniform sample of porous media is initially saturated with wetting fluid. The sample is positioned in a cup, which contains non-wetting fluid. The sample is rotated at predefined angular velocities and the displaced wetting fluid is measured at each speed. As both the wetting and non-wetting fluid during the rotation are subjected to centrifugal force a pressure gradient directed outward from the axis of rotation is created. Generally the density of the wetting fluid is higher than the one of the non-wetting fluid therefore a higher pressure is developed in the sample. This leads to an outflow of the wetting fluid at the outer radius of the sample and to an inflow of the non-wetting fluid at the inner radius. The wetting fluid is displaced by the non-wetting fluid simultaneously. At a constant rate of rotation an equilibrium saturation distribution is developed and can be determined with the capillary pressure – saturation relationship. The average saturations are obtained from the displaced volume of wetting phase depending on the angular velocity:

$$\bar{S}_w p_{c1} = \int_0^{p_{c1}} S_w(p_c)\,dp_c \,. \tag{13}$$

$\bar{S}_w$… average wetting phase saturation

P_{c1}… capillary pressure at the inner radius of the sample

Differentiating Equation 13 results in:

$$\bar{S}_w + p_{c1}\frac{d\bar{S}_w}{dp_{c1}} = S_w(p_{c1})\,. \tag{14}$$

The capillary pressure at the inner radius of the sample is determined with:

$$p_{c1} = \frac{1}{2}\Delta\rho\omega^2(r_2^2 - r_1^2)\,. \tag{15}$$

r_2… outer radius of rotation of the sample [m]

r_1… inner radius of rotation of the sample [m]

$\Delta\rho$… density difference between wetting and non-wetting phase [kg/m^3]

ω… speed of rotation [RPM]

A set of data can be obtained of a series of different speed of rotations and a plot P_{c1} versus $\bar{S}_w$ can be derived. Then the value of $d\bar{S}_w/dp_{c1}$ can be estimated from the resulting curve and these values are inserted into Equation 14 and $S_w(p_{c1})$ is computed. Then the final plot of p_{c1} versus S_{w1} can be established.

2.6 Experimental determination of imbibition capillary pressure curves

The experimental methods of Spinler and Baldwin (1997) and Fleury et al. (1999) for the determination of positive imbibition capillary pressure curves are presented and analyzed for their functionality. In this Chapter only the methods itself will be explained without any comment about the validity of these experiments. In Chapter 3.2 the problems occurring using these techniques are identified.

Spinler and Baldwin's experiment:

The aim of the experiment is to obtain positive and negative drainage and imbibition capillary pressure curves for water-wet reservoirs using the centrifuge method. With the centrifuge and the density difference between the two liquids, a pressure difference is calculated and assumed to represent the capillary pressure.

The main difficulty during a centrifuge experiment is to receive saturation information while the centrifuge rotates. In the Spinler and Baldwin's method, the oil phase is frozen while centrifuging and the water saturation is mapped with the help of a magnetic resonance imaging (MRI) tool. As non-wetting phase octadecane ($\rho = 777$ kg/m³) with a freezing point of 27 °C is used and as wetting phase de-ionized water ($\rho = 1000$ kg/m³). The ambient temperature during the experiment is 23 °C. To prevent the water from evaporation (and the octadecane from melting) the plug was kept in a sealed plastic centrifuge for the whole time.

The MRI intensity map for water was converted to water saturation with the help of a calibration curve. Little volume changes (around 2%) which arise due to the hydrocarbon contraction during freezing were adjusted with the water saturation. As soon as a uniform saturation state in the plug was reached, average MRI values were plotted against average water saturation. Depending on the core sample (sandstone or chalk) different methods are used for de-saturation. For quartz sandstone samples, porous plate and for chalk samples a centrifuge was used.

In the next step capillary pressure as a function of position in the core sample and the accomplishable pressure range was determined. This is done with the speed of the centrifuge, the length of the sample as well as the distance to the free water level. The positioning of the free water level, along the core, makes it possible to determine the positive and the negative part of the capillary scanning curves. Using a centrifuge cell with a much larger bulk volume than the one of the pore volume of the plug, the movement of the water level was minimized and can be neglected, in the experiment.

To control the direction of fluid flow and monitor the possibly occurring hysteresis effects it is necessary to prepare the plugs and define the sequence of the centrifuge steps. The plugs were sealed with Teflon on the sides so that the fluid can only enter and exit the plug at the bottom and the top. Flow is only in axial direction possible.

The following procedure is used to obtain capillary scanning curves:

1. A fully (100 %) saturated plug with wetting fluid is used.
2. The primary drainage curve is established, the centrifuge is started and the free water level is in contact with the plug.
3. The plug is prepared for the imbibition process:
   The plug is inverted (an inverted core holder is needed) and surrounded by non-wetting fluid and centrifuged. Then the plug is inverted again and the step is repeated to reach a uniform saturation profile over the whole plug at initial water saturation.
4. The plug is put in contact with the free water level and centrifuged again. The free water level is adjusted to be able to determine the positive and the negative part of the primary imbibition curve.
5. The plug is prepared for secondary drainage:
   The plug is inverted again and centrifuged in the wetting phase. Then the plug is inverted again. Repeat the step to obtain a uniform saturation profile over the whole plug at residual oil saturation.
6. The plug (in contact with the free water level) is centrifuged again. Again the free fluid level needs to be adjusted that the positive and negative part of the secondary drainage curve can be determined.
7. To obtain more hysteresis curves the steps 3 to 7 have to be repeated.

The authors claim that with this improved centrifuge method many of the limitations of a normal centrifuge experiment can be overcome or even eliminated: time of the experiment, proper shape for capillary pressure curves, saturation equilibrium and boundary conditions. However, the main goal is to obtain the negative and positive part of the drainage and imbibition capillary pressure curves, respectively.

<u>Fleury, Ringot & Poulain's experiment:</u>

Compared to standard centrifuge methods where the produced fluid (wetting or non-wetting) does not stay in contact with the core sample, in this set up the produced fluid remains in contact with the sample at all times while centrifuging, making it reversed flow possible when the pressure is decreased. Figure 5 shows the schematics of the experimental devices showing all parts necessary to establish capillary pressure curves.

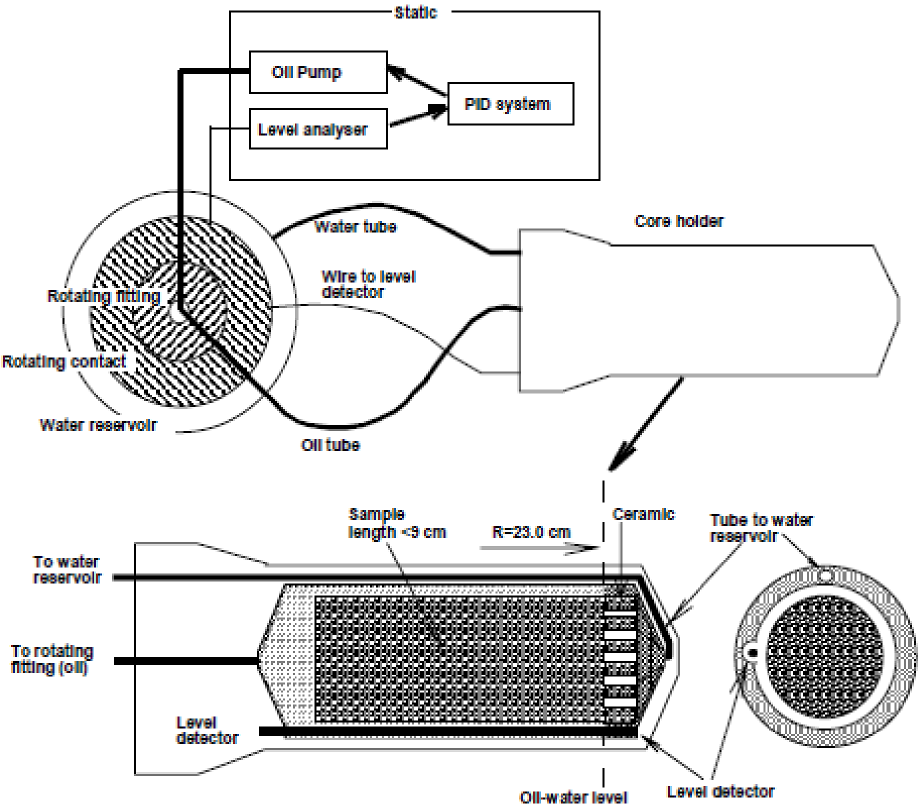

Figure 5 Schematic of the centrifuge system (Figure 2, Fleury et al. 1999). In the lower part on the left end of core holder the ceramic plate can be seen which makes it possible to establish a uniform saturation distribution after primary drainage.

Depending on drainage or imbibition the oil-water contact (in contact with the sample) is held close to the bottom or top end of the sample. Depending on the capillary pressure the oil-water contact is maintained near the bottom or top end face of the sample. For drainage and imbibition (positive capillary pressure) it is at the bottom end face. The speed is increased for drainage and decreased for the imbibition. For negative capillary pressure (forced imbibition and spontaneous drainage) the fluid contact is close to the top end face.

A pump is used to transfer oil in and out of the core holder while the centrifuge is running. Over spilled water is channeled to a tank in the middle of the rotor. With the help of a PID (proportional, integral and derivative control system) it is possible to maintain the oil-water contact constant while centrifuging. If the level moves it is recorded by the level analyzer and the pump injects/removes the necessary amount of fluid into/from the core holder to keep the level at the desired and predefined position.

The ceramic plate with a thickness of 1 cm is installed in the core holder (see Figure 5). In this experiment it is installed at a radius of 23 cm. It is a semi-permeable filter which creates a uniform saturation distribution, which was chosen to be the residual saturation, after primary drainage. The functionality of the ceramic plate is explained subsequently. The idea of the ceramic plate was first presented by Szabo (1974). A uniform saturation distribution after primary drainage makes it easier to interpret the experiment.

The procedure:

1. The sample is fully saturated with brine and put into the core holder, all parts of the centrifuge up to the rotating fitting are filled with brine. Then the centrifuge is started at a minimum speed (200 RPM).
2. The pump injects oil into the core holder, preparing for primary drainage. With the help of the detector the oil-water contact is set close to the outer face of the sample.
3. Primary drainage: The speed of rotation is increased step by step.
4. At maximum speed of 3000 RPM the oil-water contact is moved 1 cm below bottom face of the sample to obtain a uniform saturation profile within it. This is the effect of the ceramic plate, it will be explained. After stabilization the level is set back to the bottom end of the core sample.
5. Then imbibition is started by decreasing the speed of rotation in a step by step fashion.
6. Once the minimum speed of rotation has been reached, the oil-water contact is set back to the top end of the sample.
7. Now forced imbibition can be started.

There is no need to remove the core from the core holder at any point of the experiment. During the experiment data are recorded continuously. Average

saturation can be obtained from the pump. Speed of rotation gives the link to the capillary pressure, this is expressed with Equation 15.
For monitoring purpose the position of the oil-water contact, the pressure of the rotating fitting as well as the temperature inside and outside the centrifuge are recorded.

The spontaneous imbibition curve can be obtained from the production at equilibrium at each change of speed. The positive imbibition capillary pressure at the inlet of the core sample (minimum radius) is calculated using Equation 15. This equation assumes that the capillary pressure is zero at the outlet of the sample (maximum radius). The positive part of the imbibition curve is obtained in the same way as the primary drainage even if the physical processes are very different, the boundary conditions are identical and Equation 15 can be used. For deriving the secondary drainage the same procedure as for obtaining negative imbibition curves is used.

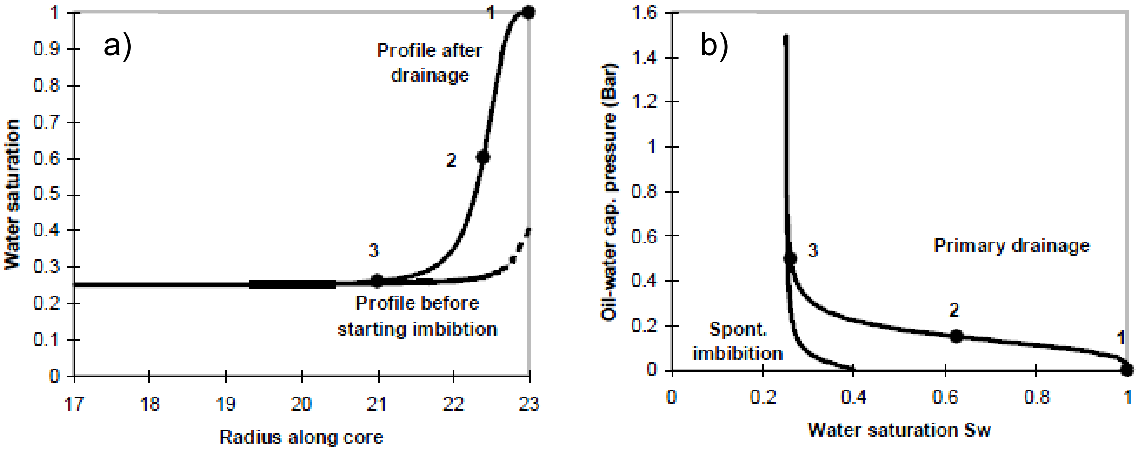

Figure 6 Effect of the ceramic plate on saturation distribution (Figure 6, Fleury et al. 1999). a): A core radius versus water saturation plot shows the original profile after drainage and profile after shifting the fluid level before starting the imbibition process. Therefore it does not matter at which point along the core the imbibition curve has its origin, the curve will look the same. b): The figure shows the same only for a capillary pressure versus saturation profile.

After primary drainage, hysteresis can occur. Fleury et al. consider that at any point on the primary drainage curve between point one and three in Figure 6 a hysteresis loop can start. With the data from this experiment, if spontaneous imbibition starts immediately after primary drainage, the hysteresis curves cannot be determined. To overcome this problem, the ceramic plate is installed at the end of the core holder (Figure 5), moving the free water level (P_c=0) one centimeter away from the core sample (to a lager radius). This moves the part between 1 and 3 (Figure 6) out of the saturation profile. The entry pressure for oil in this experimental set up is increased with the ceramic plate to around 3 bar so that no oil will flow through the ceramic plate. This leads to a uniform residual saturation profile which makes the evaluation of the imbibition capillary pressure curve much easier. In this case the imbibition capillary pressure curve is always the same, it does not matter at which point the reversal occur. Then the free water level can be set back to its original position and the outlet saturation will move back to its normal value.

3. Modelling of capillary pressure curves

As the aim of this thesis is to obtain both positive and negative imbibition capillary pressure curves in combination with a centrifuge experiment the correlation for mixed-wet reservoirs by Skjæveland et al. (1998) is used as a basis. The used correlation and the available centrifuge methods which claim to obtain imbibition capillary pressure curves are adapted and combined to a new tool.

I have developed a hysteresis scheme for capillary pressures curves in mixed-wet reservoirs. The program is modelled in Maple and combined with a tool established Ms Excel. Furthermore an artificially created centrifuge model is introduced to test the functionality of the tool.

A first code of the model was established by Skjæveland et al. (1998) as explained in Chapter 2.4. This code was reconstructed to review what has been done 15 years ago and to be able to reuse it. It constitutes the basis of this thesis project and is used to further improve work on this topic. I discovered that one part explained in the paper by Skjæveland et al. (1998) is missing. As mentioned in Chapter 2.4 an iterative process is needed to solve Equation 8 and 9. It was not possible to insert the convergence test in the old code and therefore a new code had to be established. The subsequent section explains the development of this code and the difficulties that had to be overcome.

Furthermore the two available methods to establish imbibition capillary pressure curves are analyzed with the help of my developed code. After the basic concepts have been presented, the development, the functionality as well as the limitations of the new tool are illustrated.

3.1 Base model development in Maple

Developing codes is a time intensive procedure and different models need to be established until a final model can be programmed. In this case, two examples to understand the hysteresis logic are built initially and then the final program for modelling centrifuge experiments in combination with the correlation by Skjæveland et al. (1998) is created.

All programs I developed use the same correlation by Skjæveland et al. (1998) to model capillary pressure curves and start in a similar way. First, the capillary pressure equations for drainage and imbibition, defined in Chapter 2.4, were implemented. Land's equation and the corresponding constants (residual oil saturation, irreducible water saturation, pore size distributions and entry pressures) were encoded as well. With the given equations and data, the primary draining and imbibition bounding curves are calculated as shown in Figure 7.

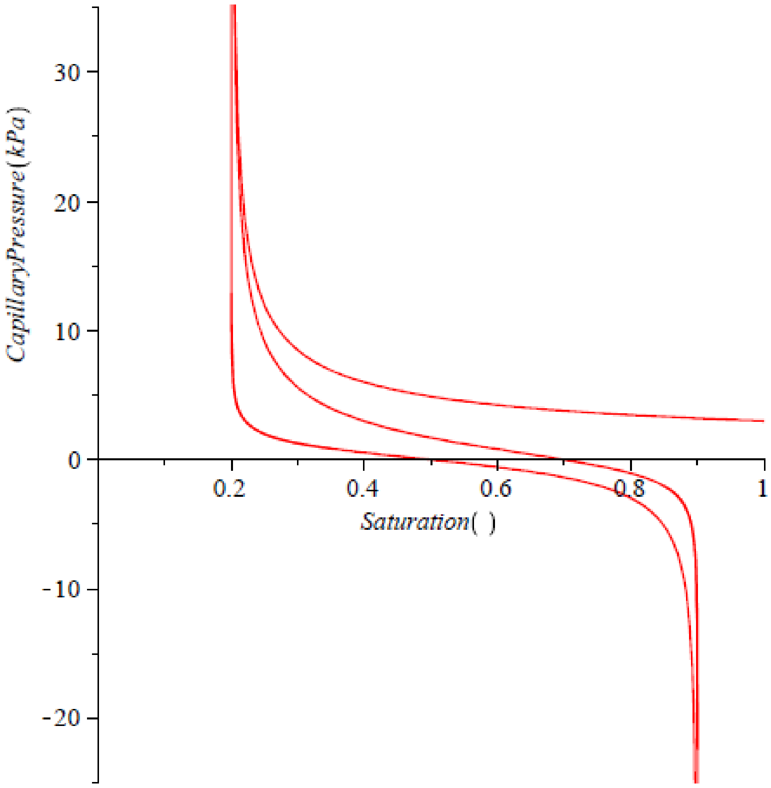

Figure 7 Bounding capillary pressure curves for drainage and imbibition. The bounding curves were created with Maple.

The first program (Example 1) was written to illustrate how reversals can be modelled. Each reversal was calculated on its own with chosen reversal points. In Figure 8 six reversals occur. The first reversal is an imbibition curve which has its origin on the primary drainage curve. The first normalized water saturation can be obtained by using the residual oil saturation for the ongoing reversals and the iteration process explained in Chapter 2.4 for the saturation values. The saturation values which are obtained by this iterative process are called minimum oil and minimum water saturation in the code. This minimum saturation is not the physical minimum saturation but a symbolic value where the two branches have this saturation value for a certain reversal. These minimum saturations can be seen in Figure 8, it is the point a reversal occurs. Point 1 in Figure 8 is the first so-called minimum saturation derived from the iteration process. The iteration process was explained in detailed in Chapter 2.4, the method developed be Skjæveland et al. (1998) was used. As already mentioned, they forgot to implement their technique in their program. I used 0.002 as the maximum deviation between two saturation values, therefore if the difference between two values is smaller, the value is approved to converge. Figure 8 shows all the reversal points and the occurring drainage and imbibition bounding and scanning curves. The entire code can be seen in the Appendix A.1.

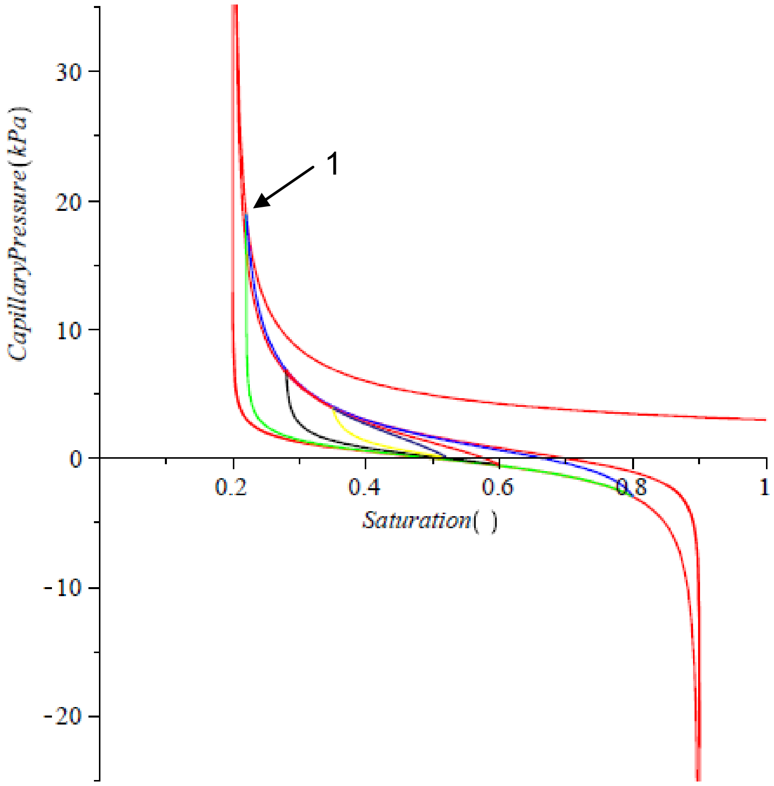

Figure 8 Example 1 of capillary pressure scanning curve modelling. Six reversals occur until the process scans back to the primary drainage bounding curve, as presented by Skjæveland et al. (1998).

For the second and third reversal two iterations are needed until the saturation values converge. Starting with the fourth reversal, three iterations are needed to reach convergence. To establish this code, saturation reversal points were predefined to generate the scanning curves. To create a more general solution, where an interval of possible saturation reversal is defined, various limitations have to be considered. The reversals points in this example were obtained by trial and error.

The second example (Example 2) uses different saturation reversal points for imbibition and secondary drainage curves. First the imbibition curve after the primary drainage curve with the origin on the primary drainage curve occurs. Then the secondary drainage starts on the imbibition curve. Therefore the imbibition curves are not ending at residual oil saturation and respectively the drainage curves are not ending at irreducible water saturation (see Figure 9). The point, the minimum saturation, where the drainage curve starts is evaluated with the help of the iteration process with Equation 8 and 9. In this code, an input interval for the saturation reversals is used and is therefore generally applicable. However, it is not possible to enforce a saturation reversal without evaluating first if it is feasible to occur at this point. Spontaneous imbibition after primary drainage always starts on the primary drainage curve. Therefore no iteration process is needed to estimate the minimum oil and water saturation as the minimum oil saturation is equal to the residual oil saturation at each reversal point on the primary drainage curve. Figure 9 shows possible imbibition capillary pressure curves after primary drainage. For the reversal points, a predefined input interval is used. The entire code can be seen in the Appendix A.2.

It is critical to find the number of necessary iterations until the minimum oil/water saturation for each drainage curve is reached. It was evaluated that every reversal needs three iteration steps until the values can be considered converging. As already mentioned, I use a predefined maximum deviation of 0.002, to check if the saturation value converges.

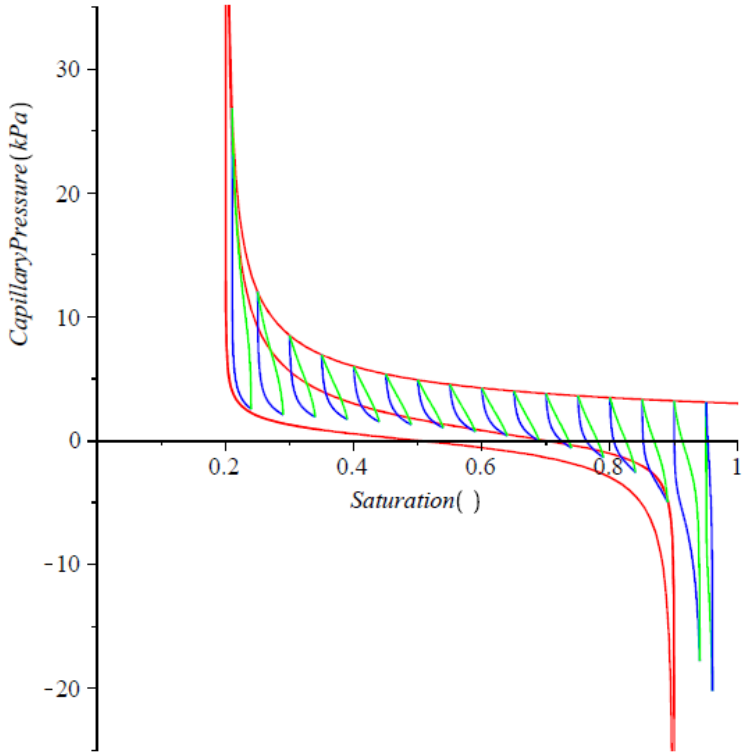

Figure 9 Example 2 of capillary pressure scanning curve modelling. Various possible imbibition curves after primary drainage and secondary drainage curves are displayed. The reversal points are predefined.

With the presentation of these programs I want to show that using the correlation by Skjæveland et al. (1998) capillary pressure curves including hysteresis effect are established and that test of convergence is important and has to be included in my program. These codes underpin my program to simulate centrifuge experiments in Maple. The established programs can be seen in Appendix A.1 and A.2.

3.2 Evaluation of experimental centrifuge methods

In Chapter 2.4 I described two commonly used centrifuge experiments in the industry to model hysteresis effects by Spinler and Baldwin (“Capillary pressure scanning curves by direct measurements of saturation”, 1997) and by Fleury, Ringot and Poulain (“Positive imbibition capillary pressure curves using the centrifuge technique”, 1999). Primarily I will discuss how the two presented papers deal with the hysteresis effect which occurs after primary drainage and why these methods are found troublesome and how I, based on the limitations of the two methods, have established new way of interpreting imbibition capillary pressure curves with the help of artificially created centrifuge data. A centrifuge experiment is simulated in Maple using Skjæveland et al. (1998) correlation for mixed-wet reservoirs.

<u>Evaluation of Spinler and Baldwin’s method:</u>

I found their method troublesome due to several reasons; I am going to explain why from my point of view this method cannot work correctly.

First of all, as it is necessary to invert the core holder after the primary drainage to start the imbibition process, the pressure continuity and the hysteresis effect are destroyed. With this experiment it is difficult to allocate where the measured imbibition curves occur along the primary drainage curve. Therefore the obtained capillary pressure curves are incorrect and the procedure cannot account for hysteresis. As mentioned by the authors, the original centrifuge method can only be used to determine the drainage or negative imbibition curves and cannot obtain scanning curves. Therefore I believe that their method does not model the occurring hysteresis effect in natural reservoirs properly. Furthermore, in this experiment only the average saturation of the core is obtained but the results are more representative if the saturation is obtained on different (predefined) points of the core. Using such an advanced method, a detailed saturation profile can be generated over the whole core.

In my option the positioning of the free water level to derive positive and negative parts of drainage/imbibition curves is questionable. In reality it is impossible to choose the position of the free water level. An experimental procedure cannot lead to representative results using techniques which cannot occur in a reservoir. As already mentioned I tried to model this procedure in Maple, but the pressure continuity is destroyed with the removal of the core. To program this method after each step, new initial data would be needed, but as there is no respective information it is impossible to establish a model. All these aspects show that this experiment cannot produce representative capillary pressure curves and lead me to a new idea. Using a centrifuge experiment in combination with a correlation to model positive and negative imbibition capillary pressure curves including the hysteresis effect. As this method was not feasible for further use, the next experimental procedure was reviewed.

Evaluation of Fleury, Ringot and Poulain's method:

They use a more solid experimental procedure and I found the procedure is comprehensive and well explained. Although hysteresis effects are considered, they try to avoid them through establishing an artificially uniform saturation distribution at residual saturation after primary drainage. As soon as the saturation distribution is uniform after primary drainage, the capillary pressure imbibition curves are identical, independent of the location. Only an average saturation is obtained since the amount of liquid pumped in and out of the sample during drainage/imbibition is compared.

Fleury et al. (1999) found a way to overcome the problem of inverting the core holder using a “Pumping While Centrifuging” (PWC) system. This system makes it possible to measure drainage and imbibition curves without stopping the centrifuge and without inverting the core holder. The pump controls the position of the free fluid level and capillary pressure curves are obtained without stopping the centrifuge and manipulation of the sample. An advantage of this system is that the produced fluid is always in contact with the core allowing the fluid to flow into and out of the sample during the process. This makes the experiment much more accurate than a normal centrifuge experiment where no contact is established.

In contrast to Spinler and Baldwin's method it is possible for me to model the method with Maple with artificially created data. I had to use artificially created data, as I did not have access to real centrifuge data, however it shows that moving the fluid level out of the sample establishes a uniform residual saturation profile after drainage. It does not matter at which position of the core the imbibition capillary pressure curve is expected to start, the curves are identical and they all start at the same reversal point which is shown in Figure 10. The Maple program to model centrifuge experiments is used. Only the boundary conditions need to be changed like it was discussed previously to use the program.

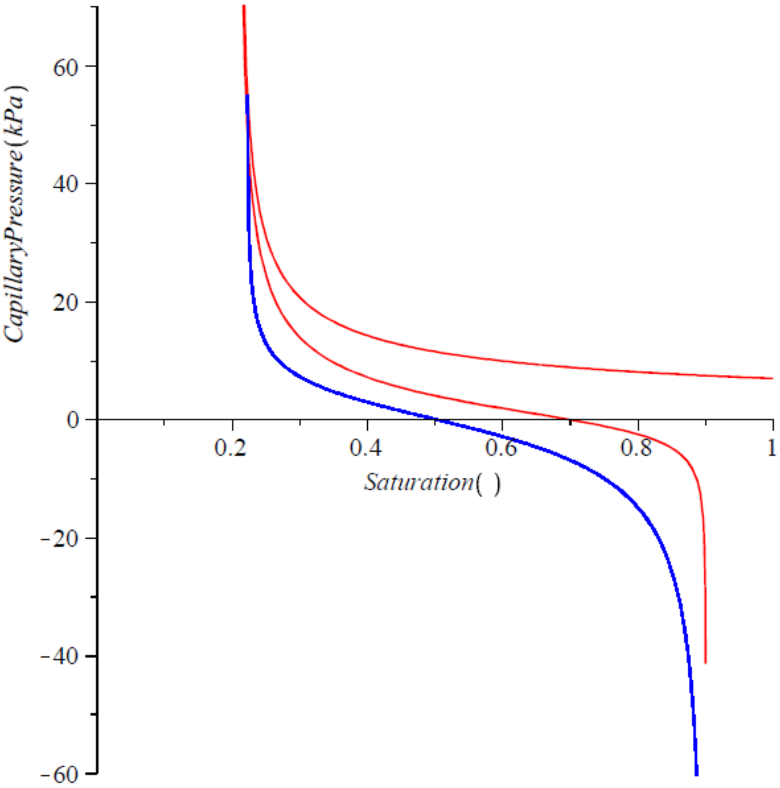

Figure 10 Imbibition curve for uniform residual saturation profile. The effect of a uniform residual saturation profile after primary drainage on imbibition capillary pressure curves is illustrated. At different positions of the sample reversal saturations and imbibition capillary pressure curves were evaluated with the result that all are identical. It is observed, if the residual saturation profile after primary drainage is uniform, the imbibition capillary pressure curve is independent of the position in the core.

In general it can be said, that the experiment from Fleury et al. (1999) is more detailed and coherently explained. The experiment is possible to model in Maple compared to the one from Spinler and Baldwin (1997). With the generated centrifuge procedure in Maple it is possible to model the effect of a uniform residual saturation profile after primary drainage (see Figure 10). It can be observed that the idea by Fleury et al. (1999) is working to produce a single imbibition capillary pressure curve. However in reality the residual saturation after primary drainage is not uniform therefore it is necessary to find a method which takes this hysteresis into account.

Following none of them found a solution to include the hysteresis scanning curves in their experiments. Therefore I want to try a new way to use a centrifuge experiment and account for hysteresis using Skjæveland et al. (1998) correlation. I simulate a centrifuge experiment in Maple (as no real saturation values are available) and combine it with the correlation to establish imbibition capillary pressure curves for mixed-wet reservoirs.

3.3 Development of imbibition capillary pressure tool

I developed a procedure which makes it possible to obtain imbibition capillary pressure curves including hysteresis and determine the residual saturations, pore size distribution indices as well as the capillary entry pressures using saturation profiles gained from a centrifuge experiments for two-phase reservoirs. I am going to explain how the presented examples in Chapter 3.1 are extended to model a centrifuge experiment. My model requires saturation data from the sample which can be obtained from a centrifuge experiment. However as no saturation data was available I created artificial saturation data from a centrifuge experiment modelled in Maple. Another reason to use artificially created data first, was to have the possibility to analyze and evaluate the tool. In this manner the correct solution to the inversion problem is known.

To establish the model I first specified the sample dimensions and the used fluids. I chose to take core dimensions from available examples by Hermansen et al. (1991). Figure 11 indicates the length of r_1 and r_2 like it was classified by Hermansen et al. (1991), where the radii are defined as followed: $r_1 = 0.0446$ m and $r_2 = 0.0938$ m. In this case the free water level has to be considered, r_2 is set to the height of the free water level which is chosen at $r = 0.093$ m.

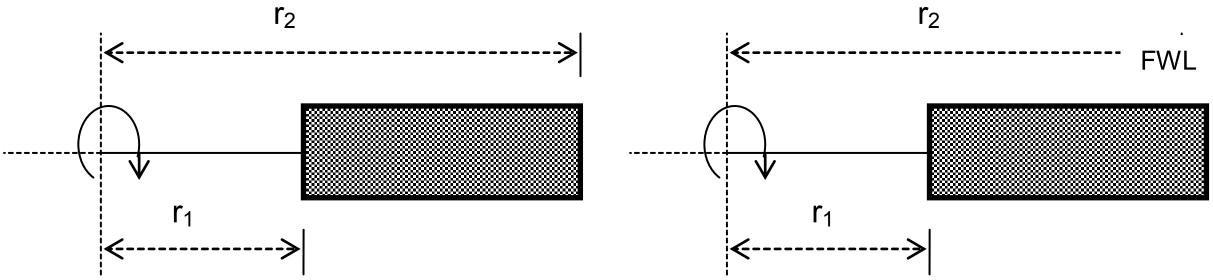

Figure 11 Schematic of a core plug. The length of r_1 and r_2 are displayed. The plot on the right accounts for the free fluid level. The one on the left does not.

The next step is to define which types of fluid are used in the centrifuge experiment. I decided to take the same fluids as Spinler and Baldwin (1997) used in their centrifuge experiment. These fluids are not ideal for a mixed-wet environment, but are the only reasonable data available and constitute a reference point to their experiment. Therefore I choose the non-wetting phase to be octadecane with a density of 777 kg/m³. The wetting phase is chosen as de-ionized water (ρ = 1000 kg/m³). For simplification the densities are assumed to be constant during the experiment. The chosen properties of the core sample and fluids (and their densities) can easily be changed in the program and have to be depending on the conducted centrifuge experiment. This makes the model adaptable to different experiments and conditions.

After defining all the input parameters, the next step is to define the radius points where reversal points, residual oil and irreducible water saturation are measured. In the beginning 16 points are used, but to establish a smooth saturation profile, the point number is increased. Depending on the capillary entry pressure, between 47 and 51 points are needed to derive representative saturation profiles. For a higher capillary entry pressure, a larger portion of the core sample is fully saturated with the wetting phase. Therefore fewer points are needed at a higher capillary entry pressure. At each point in the core, saturation and capillary pressure values are evaluated using the correlation (Equation 4) and an equation accounting for the centrifugal force (Equation 15). Then the frequency interval is chosen and the same range as for Spinler and Baldwin (1997) is used. The maximum speed is 5000 RPM (Spinler and Baldwin, 2001) and the capillary pressure and the saturation values are evaluated in 500 RPM steps.

The aim is to model a positive imbibition curve and the centrifuge starts at a speed of 5000 RPM after primary drainage. Then the angular velocity is decreased stepwise to zero. At the starting speed, the reversal saturation, the residual oil saturation and the minimum water saturation for all radii are calculated. As explained, the minimum water and oil saturation are not the physical minimum, but are variables defined in Maple. The saturation values stay constant at each point of the core and are independent of the speed of the centrifuge. Therefore the capillary pressure imbibition curve at the same position in the sample, but for different speeds has the same residual oil saturation, irreducible water saturation and reversal point. This shows that the curves vary depending on the location in the sample and again how important it is to account for hysteresis and that Spinler and Baldwind’s method cannot work, as it is not clearly illustrated at which position of the core the capillary pressure curves are measured. It is necessary to be able to determine the imbibition capillary pressure curve at each point.

The next step in the program is to define the reversal points, residual oil saturation and irreducible water saturation at each point of the core. Two different equations for capillary pressure are used to solve for the parameters. The correlation

(Equation 4) for mixed-wet reservoirs by Skjæveland et al. (1998) is used and combined with Equation 15 which accounts for the centrifugal force (Fleury et al. 1999). The three parameters (reversal point, residual oil saturation and irreducible water saturation) are evaluated at every desired location (r). They are used to determine the imbibition capillary pressure and the saturation values over the whole core sample. Moreover a saturation profile and the imbibition capillary pressure curves are obtained at every point of the core sample and therefore accounting for the hysteresis effect.

This was done for two different data sets as I first had to create artificial saturation data. After both runs are completed, a way of adjusting the correlation parameters to fit the measured values had to be found. Therefore I considered the three following methods:

- method of the steepest descent
- nonlinear solver
- Maple as an Add-in in Excel

I used Ms Excel with its nonlinear solver as it was the best solution for me. It is a quick method to develop a well arranged and comprehensible tool. Even the solver assumes linear independent variables, it was used to evaluate the procedure. The method of steepest descent would be more exact but it also requires professional programming skills as well as more time as six parameters have to be adjusted. Another possibility is to use the so-called “Maple an Add-in” in Excel. This Add-in makes it possible to use Maple commands within Excel. This option was tried as well, but it gets impossible to retain an overview as the equations become too complex and long due the implemented of Maple Add-in in Ms Excel is complicated.

Using the Ms Excel solver, the error between the values of the saturation obtained from the “test data” and the “composed data” is calculated. Then the total error is calculated and is minimized by adjustment of the parameters by the Ms Excel solver. To use the solver the saturation values of the “composed data” have to be generated in Ms Excel itself. An equation using the parameters is necessary, that the solver can be used to adjust the parameters for another solution (in this case to the saturation “test data”). To calculate the saturation values with the assumed parameters, a Macro is written in Visual Basic, which makes it possible to generate the “composed saturation values”.

The used Macro called “ComputeS” (see Appendix A.5) solves for the saturation values from the correlation (Equation 4). The values for the parameters should be selected based on the capillary pressure and saturation data from the centrifuge. A reasonable judgment should be made. However the parameters have to be the same used in Maple to generate the capillary pressure values. The Macro “ComputeS” uses the bisection method. The bisection method solves iteratively for the saturation value at each point of the core sample. The maximum number of iterations and the tolerance are defined in the Excel tool and can be adjusted.

The squared error between the two saturation values is calculated and summed up to the total error. The total error is needed to use the solver. The Ms Excel solver is a regression routine using the generalized reduced gradient method to solve for non-linear problems (Microsoft, 2013). Therefore the solver iterates to minimize the total error by changing the parameters (in this case, residual oil and irreducible water saturation, pore size distribution indices and the capillary entry pressures for both phases) independently until the minimum error is found. It is a simple and quick method with the accuracy and the results depending on the frequencies, discussed in the result section.

The saturation profiles of the computed saturation are plotted before and after solving for the parameters and compared with the one obtained from the test data. This makes it possible to visually evaluate the curve fitting.

For a better understanding of the procedure the steps are explained in detail:

1. An artificially “test data” set is created (if saturation data is available, this step is not needed):

   The centrifuge experiment is used (Appendix A.3) to establish capillary pressure and saturation data. Therefore the values presented in Table 1 for residual oil saturation, irreducible water saturation, the pore size distribution for oil and water and the capillary entry pressure for oil- and water are used.

a_{wi}	0.25	c_{wi}	10.0
a_{wd}	0.50	c_{wd}	7.00
a_{oi}	0.50	c_{oi}	-8.56
a_{od}	0.25	c_{od}	-6.10

Table 1 Values of the parameters to generate the test data. The presented values are the proposed values from Skjæveland et al. (1998) for mixed-wet reservoirs.

The artificial centrifuge test data is established and will be called “test data” in the further work.

2. For the second run, normally the first procedure step, random values (were the user of the tool has the opinion are a good match to achieve the obtained capillary pressure versus saturation curves from the centrifuge experiment) are chosen for the parameters and the model (Appendix A.3) is ran again and new imbibition capillary pressure and saturation values are obtained. This data is called “composed data” in the further work.
3. After all saturation data is available the data is exported to the Excel tool (see Appendix A.4).

   The saturation values obtained from the “test data” set only are exported from Maple and imported into Excel. The data has to be written into the column “Input saturation” corresponding to the right radius which is entered in the column “r”. If the data is obtained from a real experiment, the data has to be entered in the Excel tool as explained.

   From the “composed data” set, the capillary pressure at each radius is found and exported to Excel. The capillary pressure values have to be entered in the column called “Input Pc”. There is no need to export the saturation values as the saturation is calculated using a Macro written in Visual Basic called “ComputeS” using the assumed parameters to calculate the values in the column “ComputedS”. The capillary pressure versus saturation is plotted, one curve for the “test data” and one for the “computed data”.
4. After all the data is available in the Excel tool, the evaluation of the parameters (residual oil saturation, irreducible water saturation, the pore size distribution for oil and water and the capillary entry pressure for oil and water) is next. This is done using the Excel solver. The Excel solver is started and the parameters are evaluated. Furthermore the final capillary pressure curve is plotted and is compared with the one obtained from the “test data”. This makes it possible to visually evaluate the curve fitting.

Assumptions and Limitations:

- Using the correlation from Skjæveland et al. (1998) the same assumptions are adopted. They are presented in Chapter 2.4.
- Artificially created saturation data is used to test the functionality of the tool
- The density of the fluids is constant during the centrifuge experiment
- The used non-wetting fluid (octadecane) is not ideal for mixed-wet reservoirs
- Ms Excel solver treats input variables as linearly independent
- No physical boundaries for the parameters (residual oil saturation, irreducible water saturation, the pore size distribution for oil and water and the capillary entry pressure for oil and water) are entered

To reasonably develop the tool, some assumptions had to be made considering the extent of this project. This tool represents a basis for capillary pressure modelling in mixed wet reservoir and further improvements are proposed.

4. Comparison of model output and results with test data

It will be demonstrated that with the presented centrifuge model, saturation profiles and imbibition capillary pressure curves in mixed-wet reservoirs can be generated. The residual saturations, pore size distribution indices, capillary entry pressures and for each point along the core, an imbibition capillary pressure curve are obtained. The full model is presented in the Appendix A.3, the Excel tool in the Appendix A.4 and the Macro in the Appendix A.5.

All the displayed outputs of Maple are produced from the same code, the “test data” is used to show the results produced from Maple. Figure 12 shows the saturation profiles of the “test data” at different speeds of the centrifuge.

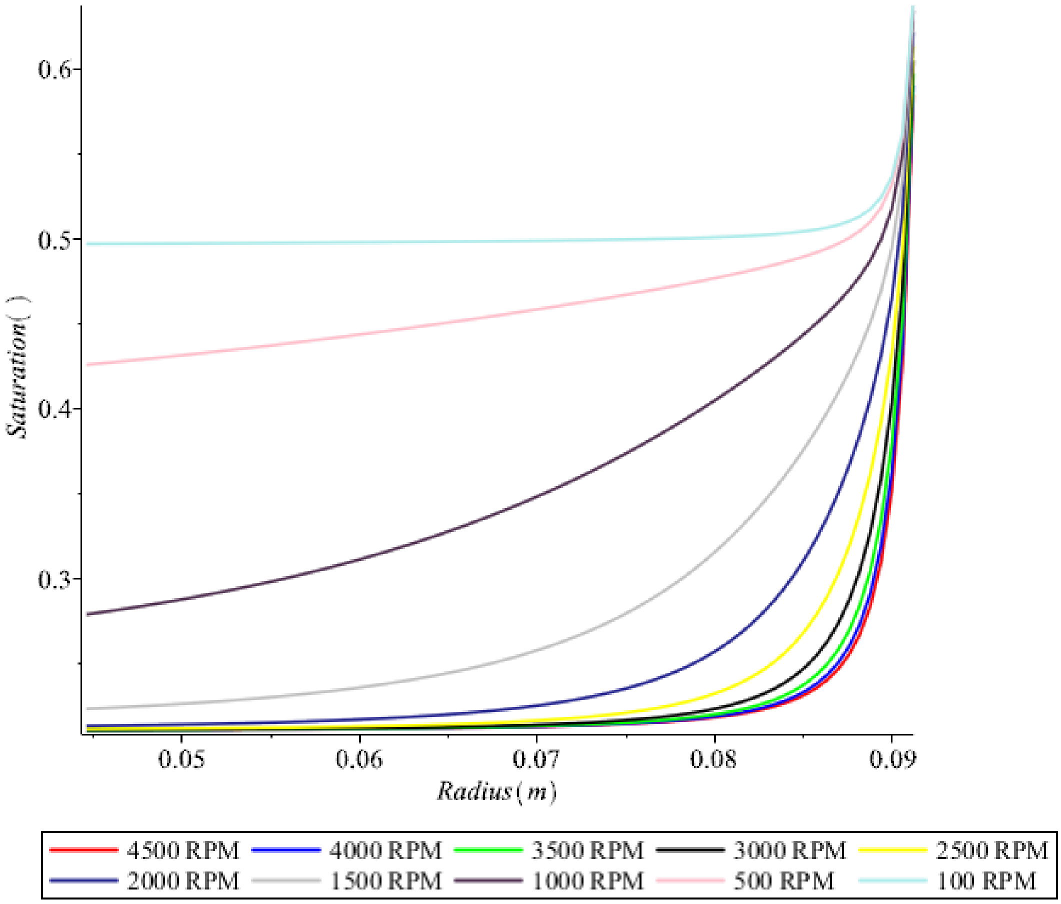

Figure 12 Saturation profiles of the test data. Each color represents the saturation profile at a different speed. The red curve is at the highest speed (ω = 4500 RPM) and the turquoise one for the lowest speed (ω = 100 RPM). It can be seen, that the water has fully filled the core at a speed of 100 RPM.

In Figure 12 it can clearly be seen that the saturation is a strong function of the radius.

The saturation profiles at the higher speeds are becoming close and the profiles less dependent on a further increase in speed. Figure 13 shows the saturation profiles for speeds between 1500 RPM and 4500 RPM in a detailed in view.

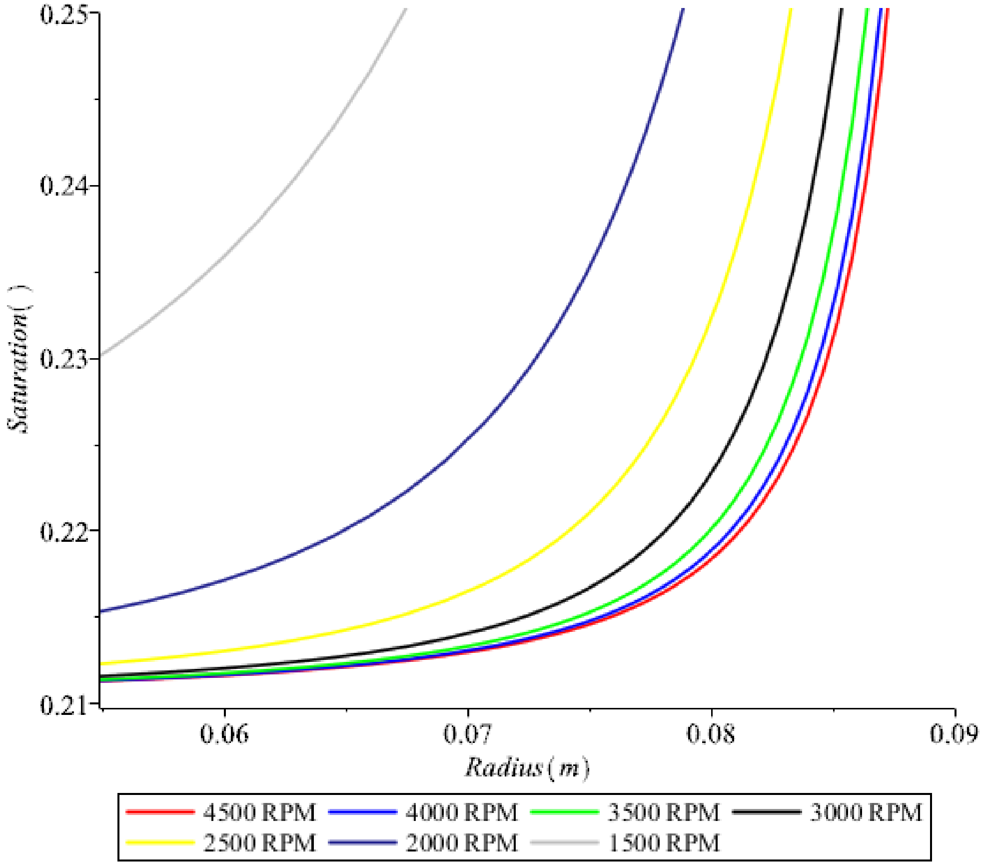

Figure 13 Saturation profiles of the test data in a more detailed view. In this figure a magnified view of the core sample from Figure 12 is shown. Now the saturation profiles at the higher speeds can be seen as well.

In the region of the curvature, the density of the points is increased to have better resolution and establish a smooth curve.

From Figure 12 and Figure 13, it gets obvious that the saturation profiles are really close together at higher speeds. A curve indicates the movement of the fluid between each speed reduction. At higher speeds the core is mainly saturated with non-wetting phase and the water imbibes slowly into core. At a speed of 3000 RPM more and more water is sucked into the core and finally at a speed of 100 RPM the core is mainly water saturated.

The saturation profiles do not give any information about the hysteresis effect and therefore the capillary pressure profile has to be observed. In Figure 14 the capillary pressure curves of the primary drainage and imbibition are shown. It can be seen that the reversal for an imbibition curve can be at every point and that the imbibition curves can be determined using this model at every point. From Figure 14 it can be observed that the residual oil saturation, the reversal points and the irreducible water saturation for the different imbibition capillary pressure curves varies depending on the point of the core, as it was explained above.

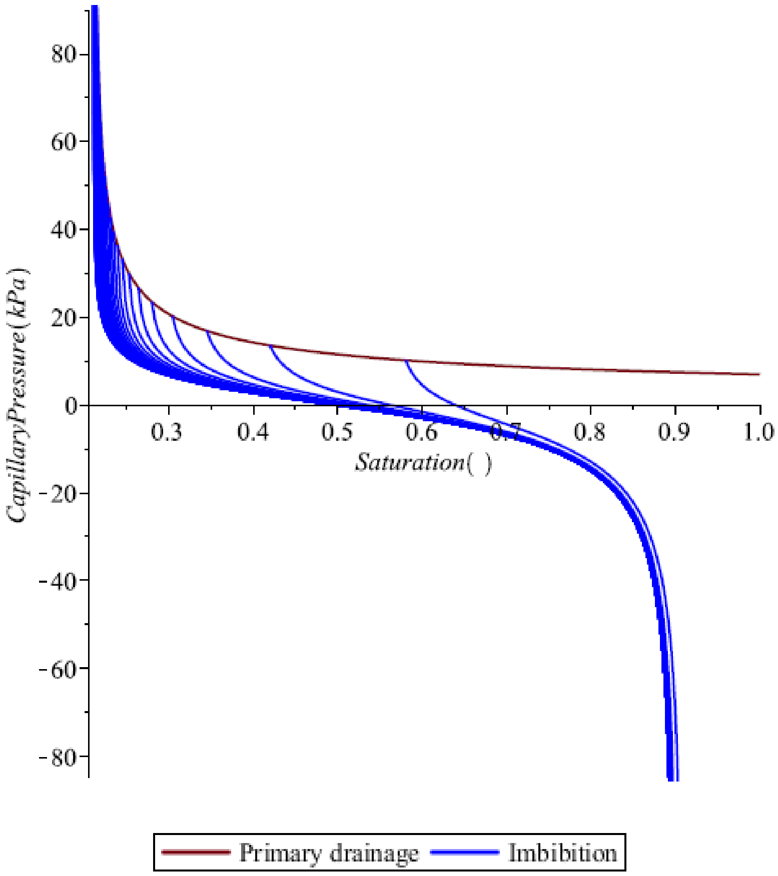

Figure 14 Capillary pressure curves of the test data. The red curve shows the primary drainage and the blue ones indicate the imbibition curves. Each reversal point is calculated with the chosen interval over the core.

As the reversal saturation point depends on the point of the core where it is determined, most of the imbibition curves start close to the irreducible water saturation. Many points in the core sample are close to irreducible water saturation as in this area a lot of radius points are needed to create a smooth saturation profile as already mentioned. It is possible to define even more imbibition curves, depending on the density of the chosen interval over the core. The location and the number of saturation values have to be selected for each single experiment

Figure 15 shows the saturation range from 0.2 to 0.3. In this area, the imbibition curves do all have different origins on the primary drainage curves, which can hardly be seen in Figure 14 where it looks like they have the same origin.

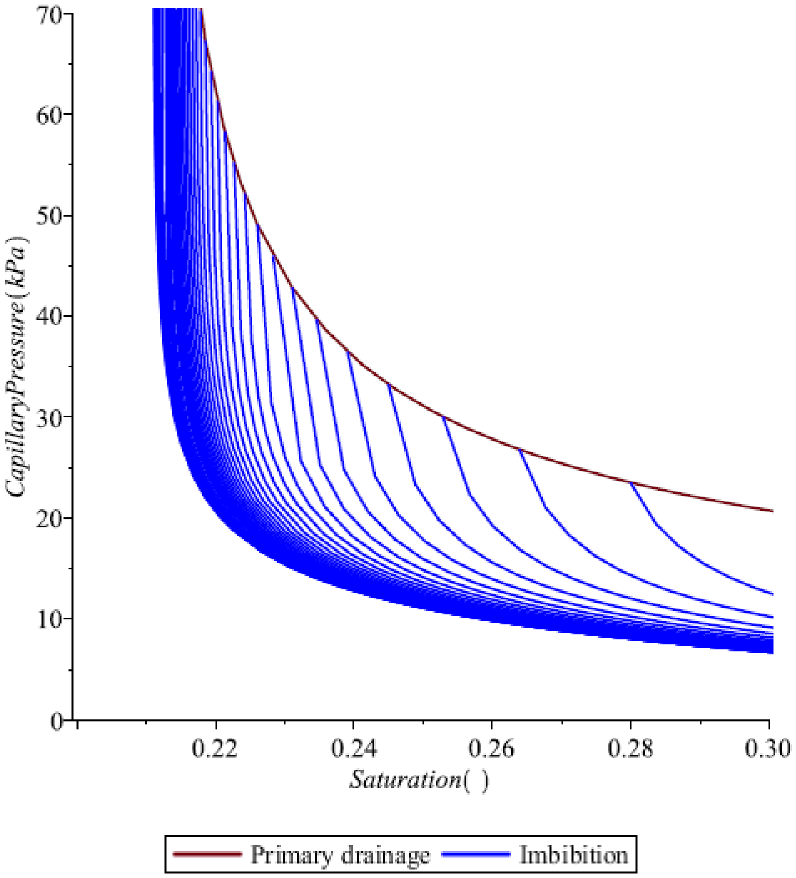

Figure 15 Capillary pressure curves of the test-data in a detailed view. It can be seen that all imbibition capillary pressure curves are different even if their origin on the primary drainage curve is really close.

The presented curves are generated with the test data and according to the varying input data the shape of the curve varies but the principle is the same.

As the test data set was created, two different data sets representing the calculated data are chosen. Different data sets are compared to test the curve fitting tool within Excel. Two cases are shown one with higher values and one with lower values than the original parameters.

Any first estimate is run in Maple, and then the estimated values and the capillary pressure values are exported to Excel. If the values differ too much from the original data, the solver cannot find a solution. If the fit looks good and the curves are similar, the solver can be applied (Figure 16). Though if the curves are too different it is better to adjust the parameters first and as soon as a better fit is found, the new parameters are used again in Maple to produce new capillary pressure values. Then the new capillary pressure values are exported to Maple again and the curve fitting can be started. Figure 16 shows in which case the curve fitting works and in which not.

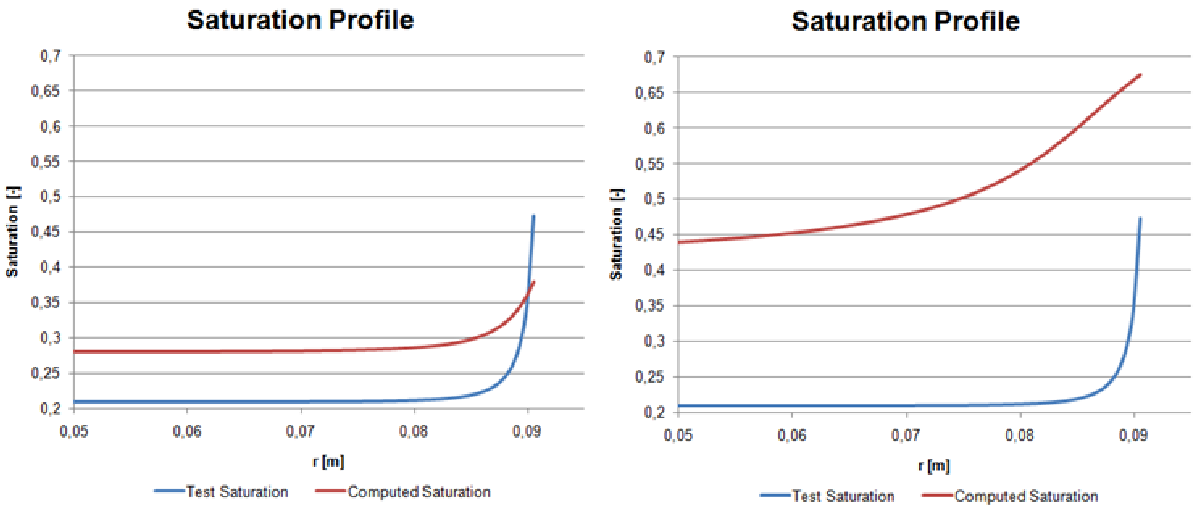

Figure 16 Comparison of two different saturation profiles. The saturation profile on the left side is from Case 1 (higher case) at a centrifuge speed of 3500 RPM. The saturation profile on the right side is at the same speed but without adjusting the parameters (the adjustment is explained previously). The first estimate of the parameters was too far away from the original test data and therefore the Excel solver cannot solve for it. After adjusting the parameters to reach a smaller difference between the red and the blue curve (left graph), the curve fitting process works without any problems.

The final values for running the curve fitting and obtaining the capillary entry pressures, pore size distribution indices and saturations are summarized in Table 2.

For a better evaluation of the tool, I decided to create two different data sets called Case 1 and 2. In Case 1, I choose slighter higher and in Case 2 a little bit lower values for the parameters than the one used to create the test data. I select these two cases to determine whether there is a difference if the values are higher or lower than the real ones.

	Unit	Test data	Case 1	Case 2
a_{wi}	[-]	0.25	0.30	0.20
a_{oi}	[-]	0.50	0.60	0.40
a_{wd}	[-]	0.25	0.30	0.20
a_{od}	[-]	0.50	0.60	0.40
c_{wi}	[kPa]	10.00	11.00	8.00
c_{wd}	[kPa]	7.00	8.00	5.00
S_{or}	[-]	0.10	0.13	0.10
S_{wr}	[-]	0.21	0.28	0.18
S_1	[-]	0.50	0.50	0.50
S_2	[-]	0.70	0.70	0.70

Table 2 Input parameters for test data, Case 1 and 2. Summary of the parameters entered in Maple and the set used for the imbibition are also used for the excel file.

It can be noticed that the capillary entry pressures for the oil phase are missing. They are determined using the correlation and the points S_1 and S_2. These are crossover points respectively from imbibition to drainage and from drainage to imbibition on the x-axis (saturation-axis). Solving Equation (4) for c_{oi} leads to:

$$c_{oi} = -\frac{c_{wi}\left(\frac{-1+S_1+S_{or}}{-1+S_{or}}\right)^{a_{oi}}}{\left(-\frac{S_1-S_{wr}}{-1+S_{wr}}\right)^{a_{wi}}}, \tag{16}$$

and accordingly for c_{od} S_1 is substituted by S_2 and all subscripts "*i*" by "*d*". This method is used in the Maple model as well. The crossover point is chosen to solve for the capillary entry pressure for the non-wetting phase. After the saturation data is evaluated after every 500 RPM, the test data and composed data are compared and the minimum error between the two data sets is evaluated with the Excel solver by adjusting the parameters. The results of the two cases will be discussed subsequently.

Case 1:

In this case the input parameters are higher than the original test data. In Table 3 the original values (used to generate test data) and the results of the parameters after running the solver at each speed are displayed. The used guess can be seen in Table 2.

	RPM **orig.**	**500**	**1000**	**1500**	**2000**	**2500**	**3000**	**3500**	**4000**	**4500**
S_{wr}	0.21	0.135	0.201	0.216	0.219	0.215	0.213	0.212	0.210	0.210
S_{or}	0.10	0.146	0.135	0.050*	0.078	0.117	0.099	0.090	0.213	0.221
a_{wi}	0.25	0.162	0.265	0.260	0.091	0.157	0.142	0.149	0.213	0.214
a_{oi}	0.60	0.570	0.221	0.623	0.554	0.586	0.625	0.646	0.641	0.660
c_w	10.0	11.00	10.99	11.00	10.99	10.99	10.96	11.00	11.00	11.00
S_1	0.50	0.506	0.526	0.593	0.516	0.595	0.664	0.731	0.646	0.714

Table 3 Results of the parameters at different angular velocities for Case 1. The results from case one (higher case) are embraced in the table to compare them. * In this case, the program chooses S_{or} to be zero, as the residual saturation will never be zero, a restriction was given that it has to be greater than 0.05.

For the irreducible water saturation, the tool estimates almost the same values except for speeds lower than 1000 RPM. The tool does not always yield the same results as for example for the residual oil saturation different values are obtained at each speed. It is similar for speeds of the centrifuge between 2000 RPM and 3500 RPM. For the pore size distribution index, the evaluation tool leads in some cases to close results compared to the original values and in some slightly different. For the wetting phase, the parameters are quite similar at low and really high speeds of the centrifuge. For the oil phase it is roughly the same except for a speed of 1000 RPM. Having a view on the results for the capillary entry pressure, it is conspicuous that they are very similar compared to the input parameter. The tool does not change the parameter to minimize the total error between the data sets.

For determining the crossing point and therefore the capillary entry pressure for the non-wetting phase, the highest variations can be noticed especially as the same value was used for the test and the calculation data. It can be observed that the values tend to increase with an increase of the speed of the centrifuge. To make a judgment if the tool works for curve fitting purpose the saturation profiles are visualized in Figure 17.

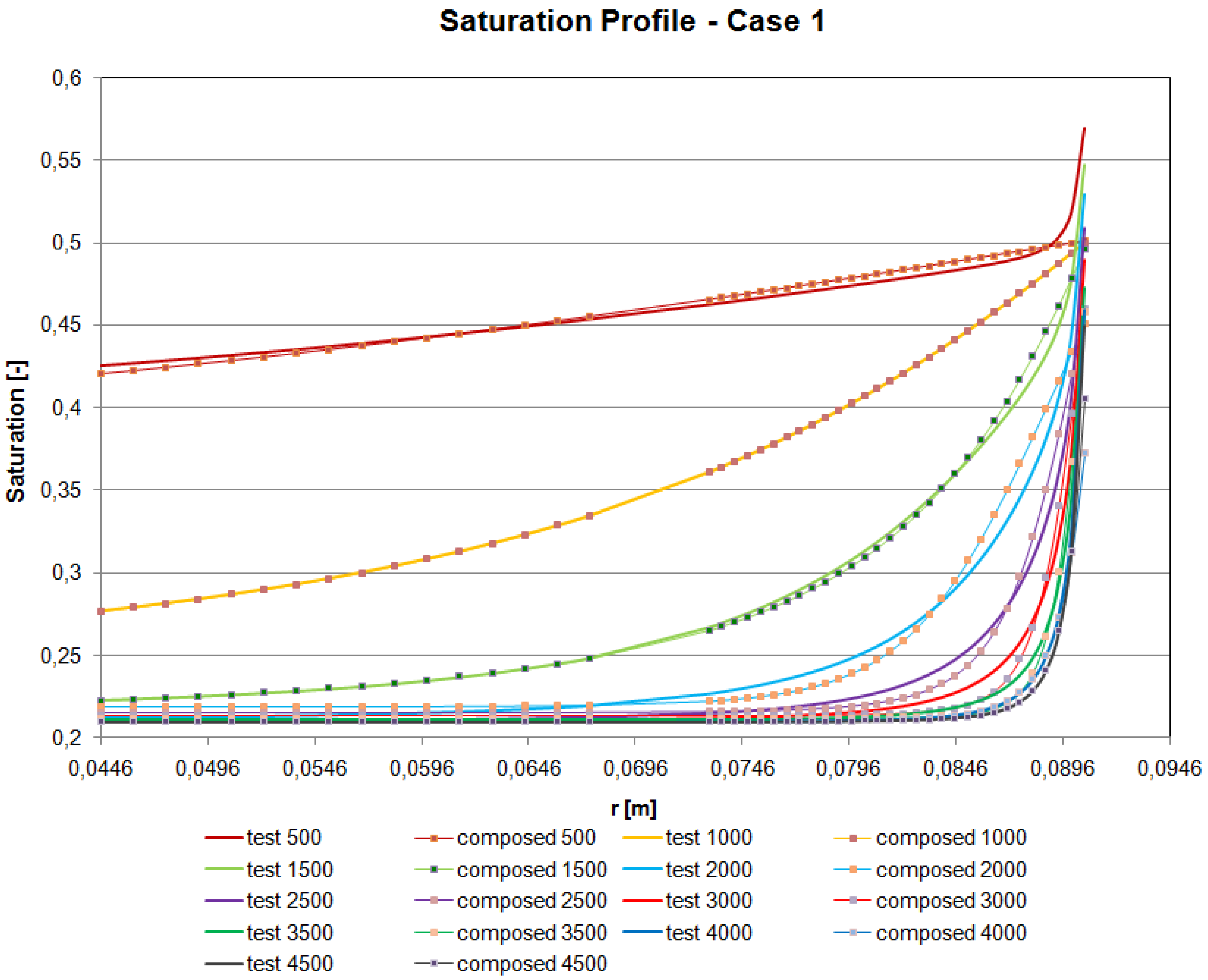

Figure 17 Comparison of test to composed saturation profiles Case 1. All saturation profiles for both the test data case and the adjusted calculated data are displayed. The curves for the same speed are displayed in the same color. The thicker line without points represents the test data and the other line with dots the calculated data.

Figure 17 shows the produced saturation profiles from the test data compared to the saturation profiles obtained by the centrifuge model and the Excel tool. The dark red curves are at the lowest speed of 500 RPM where almost the whole wetting phase is already sucked into the core.

In general, the visual fit looks accurate with some deviations that can be seen. At some speeds both curves are almost identical, for example at a speed of 1000 RPM and at high speeds of 4000 RPM and 4500 RPM. Checking the deviation of the values the match for the speed of 1000 RPM is quite good as well except for the value for the non-wetting pore size distribution index. At higher speeds the calculated data is not matching the irreducible non-wetting saturation and the capillary entry pressure of the non-wetting phase. Still, the saturation profile looks good.

Case 2:

In this case the used input parameters are lower than the original test data. In Table **4** the original and the parameters after running the solver at each centrifuge speed are displayed. The estimate can be seen in Table 2.

	RPM orig.	**500**	**1000**	**1500**	**2000**	**2500**	**3000**	**3500**	**4000**	**4500**
S_{wr}	0.21	0.404	0.191	0.202	0.216	0.215	0.213	0.212	0.210	/
S_{or}	0.10	0.096	0.170	0.138	0.105	0.106	0.106	0.087	0.179	/
a_{wi}	0.25	0.287	0.602	0.450	0.245	0.222	0.210	0.213	0.251	/
a_{oi}	0.60	0.413	0.298	0.309	0.386	0.435	0.412	0.499	0.654	/
c_w	10.0	8.003	7.899	8.007	7.998	7.977	8.002	8.009	8.019	/
S_1	0.50	0.527	0.539	0.584	0.608	0.678	0.770	0.834	0.762	/

Table 4 Results of the parameters at different angular velocities for Case 2. The results from case two (lower case) are embraced in the table to compare them. "/" means it was not possible to evaluate the parameters at this speed.

Evaluating the results in Table 4, the values are quite similar to the first case. The same abnormalities can be observed for each parameter. Again, the values for wetting phase capillary entry pressure are fairly constant, the value for the crossing saturation point (therefore non-wetting phase capillary entry pressure) tends to increase with increasing rotation speed and the most consistent values are obtained for irreducible water saturation.

At a speed of 4500 RPM, the tool has troubles although it is possible to determine values for the parameters, the Macro "composeS" is not able to determine the saturation values anymore. The problem occurs because the curve is really sharp and the tool has troubles to determine the values at these points. This case is visualized in Figure 18 and the curve fitting can be observed.

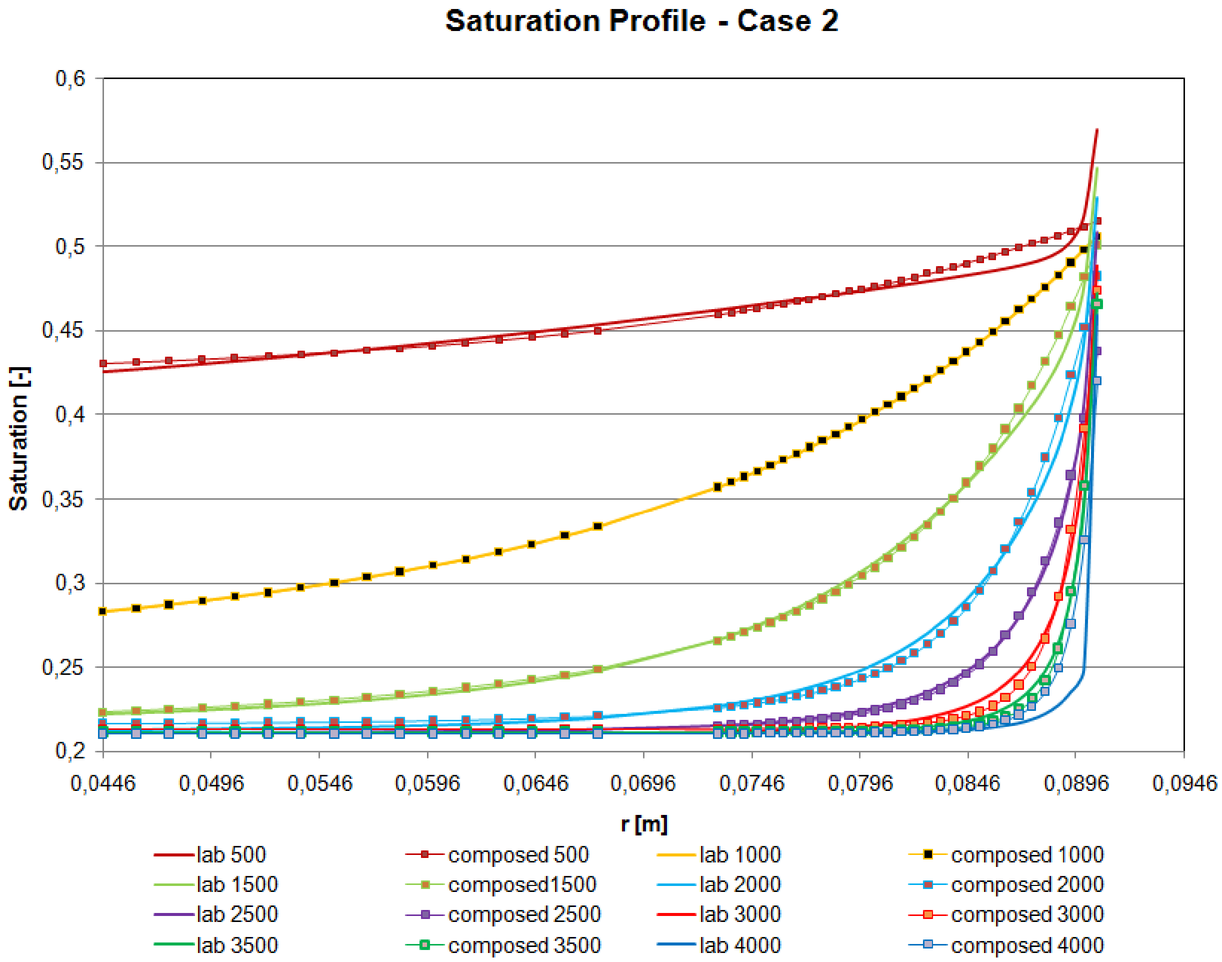

Figure 18 Comparison of test to composed saturation profiles Case 2. All saturation profiles for both the test data case and the adjusted calculated data are displayed. The curves for each speed are displayed in the same color. The thicker line with no points represents the test data and the line with dots the composed data.

Looking at Figure 18 most fits show minor deviation. At speeds of 1000 RPM, 2500 RPM and 3500 RPM it achieves most exact matches. Comparing the obtained parameters of these speeds to the test data, it looks like the fit for 1000 RPM does not fit really well. The deviations for the both pore size distribution indices are higher as well as the obtained residual non-wetting saturation. The fit for 2500 RPM is better, still the pore size distribution index for the non-wetting phase is too low and as the crossing point saturation is increasing with speed it gets too high. At 3500 RPM, the match is fairly good, only for the crossing saturation point a high difference can be noticed which leads to a change in capillary entry pressure for the non-wetting phase. A direct comparison of the deviation of each parameter compared to the used one to create the test data are shown in Table 5 and Table 6.

		500	1000	1500	2000	2500	3000	3500	4000	4500
S_{wr}	[%]	35.73	4.46	3.83	4.46	2.42	1.63	0.96	0.11	0.09
S_{or}	%	45.92	34.73	50.00	22.04	16.51	1.03	9.79	87.17	120.70
a_{wi}	%	35.38	6.06	89.18	64.07	37.19	43.08	40.45	14.78	14.12
a_{oi}	%	5.00	63.24	1.17	7.76	2.33	4.18	7.72	6.90	9.93
c_w	%	9.96	9.85	10.13	9.91	9.88	9.63	10.01	9.99	9.98
S_1	%	1.24	5.22	13.87	2.96	18.99	32.83	46.17	29.14	42.77

Table 5 Deviation from the computed parameters to the test parameters Case 1. The deviation of each parameter for the first case referring to the used values creating the test data is shown.

		500	1000	1500	2000	2500	3000	3500	4000	4500
S_{wr}	%	92.29	9.19	3.81	2.62	2.24	1.39	0.76	0.11	/
S_{or}	%	3.71	69.52	38.35	5.25	2.55	6.26	14.42	79.41	/
a_{wi}	%	14.87	140.99	80.39	1.88	11.48	16.01	14.62	0.54	/
a_{oi}	%	31.17	50.25	48.44	35.70	27.50	31.27	16.86	9.07	/
c_w	%	19.97	21.01	19.93	20.01	20.23	19.98	19.91	19.80	/
S_1	%	5.41	7.75	16.85	21.51	35.53	54.03	66.73	52.41	/

Table 6 Deviation from the computed parameters to the test parameters Case 2. The deviation of each parameter for the second case referring to the used values creating the test data is shown.

It can be seen that the deviation varies from 0.11 % to 140 %. Extreme cases appear at the high speed or at low speed of the imbibition process. For example the highest deviations for irreducible wetting phase saturation is always at a speed of 500 RPM and except of one speed for all other speeds for both cases the deviation is less than 5 %. The input parameters are equal to 0.18 and 0.28 (see Table 2) and for both cases the S_{wr} is really close to the original input parameter.

The comparison between the visual and the numerical results should show that both results have to be evaluated to get a better understanding of the results. As the curve fitting is based on a mathematical (generalized reduced gradient) iteration method to find the smallest error, the parameters are not adjusted based on physical reasoning (for example in Case 1 S_{or} is adjusted to zero, which is physically not possible). The parameters are adjusted to find the smallest error which is shown to generally lead to reasonable results.

The parameters are obtained for each speed separately and no general solution for one case is evaluated. It would be desirable to create a tool which considers the results of all speeds and try to find one best fit for all curves. For now an average of each parameter is calculated for each case and the results are shown in Table 7.

	Case 1	**Case 2**
S_{wr}	0.20	0.23
S_{or}	0.12	0.12
a_{wi}	0.21	0.31
a_{oi}	0.57	0.43
c_w	10.99	7.99
S_1	0.61	0.66

Table 7 Averaged parameters for Case1 and 2. For establishing parameters for one case and not for each speed the average of each parameter is taken.

The average of the estimated parameters results in a reasonable solution. Case 1 is really close to the chosen input parameters. Even if the average values for the second case are slightly higher, the results before showed that the fit is really good.

The summary of the results should give an overview of the possibilities of this tool and what can be done with it. The usage and possible improvements will be evaluated in the discussion.

5. Discussion

The importance of finding evaluation methods for capillary pressure curves has been highlighted in the introduction. With the correlation by Skjæveland et al. (1998), a way to model and establish capillary pressure curves with consideration of the hysteresis effect is available. In addition, the determination of residual saturations, pore size distribution indices as well as capillary entry pressures is essential in the context of capillary pressure modelling and can be performed with the designed tool.

It is shown that the widely used method by Spinler and Baldwin (1997) to derive capillary pressure curves does not reach their claimed results. The detailed evaluation leads to the conclusion that with this technique, continuous capillary pressure curves cannot be produced. Likewise the hysteresis effect is not considered in the method and the use of the method leads to incorrect capillary pressure curves. The alternative method introduced by Fleury et al. (1999) uses a uniform saturation profile at residual saturation to avoid hysteresis. Figure 10 shows that with a uniform residual saturation profile after primary drainage all imbibition curves will be identical and start at the same point on the primary drainage curve.

With the limitations highlighted it is shown that no reasonable experimental way to establish imbibition capillary pressure curves for two-phase is available. Hence, a new method is proposed where the combination of a centrifuge experiment and the capillary pressure correlation are used to establish hysteresis imbibition capillary pressure curves as well as to derive the residual saturations, capillary entry pressures and pore size distribution indices for both phases.

The proposed model has been developed using artificially created “test data” to test the procedure and its functionality. Based on the established model as a follow up, it is recommended to test the method with real saturation profiles obtained from a centrifuge experiment with the known limitations.

The development of the tool has led to findings to overcome some of the assumptions stated in Chapter 3.3, first of all the use of the method of steepest decent instead of an nonlinear solver. Then it would be possible to account for the linearly dependence of the parameters therefore more exact result can be obtained and the value for the capillary entry pressure will be adjusted as well. Furthermore I figured out, that octadecane should probably not be used as non-wetting fluid in a mixed-wet reservoir, but it was the only reasonable data available and constitute a reference point to Spinler and Baldwin (1997). Besides it is difficult to find centrifuge experiments which were conducted in a mixed-wet reservoir. Moreover I assumed for simplicity that the densities of the used fluids in the centrifuge stay constant during the process. An improvement would be to vary the density with the angular velocity of the centrifuge.

Furthermore it would be desirable to enter boundary conditions for the parameters (saturation values, pore size distribution indices and capillary entry pressures) and make the tool more automatic. Suggested add-ins would be on the one hand that the program could tell if a parameters deviation is too high to find a curve match with the solver. On the other hand it would be helpful to set boundaries for the tool that a parameter stays within physical limits and does not converge to a mathematical fit which is unreasonable.

6. Conclusion

To incorporate mixed wettability, recently used methods are reviewed to find a way to determine imbibition capillary pressure curves after primary drainage. It was explained that most available techniques are especially troublesome due to the occurring hysteresis effect. It was discussed that the available experiments produce incorrect imbibition capillary pressure curves as hysteresis effect is not considered or artificially removed. These experiments should not be used as they lead to inaccurate results which have a major influence on predicting flow behavior and well performance. However it is important to find a way of modelling imbibition capillary pressure curves. Therefore this new method, a combination of a centrifuge experiment and a correlation for capillary pressure is used. The correlation by Skjæveland et al. (1998) was chosen as it is comprehensive and widely used.

With the presented tool, not only capillary pressure curves can be obtained. To derive capillary pressure curves with the correlation by Skjæveland et al. (1998) the residual saturations, pore size distribution indices and capillary entry pressures need to be known. If the saturation profile of a core obtained by a centrifuge experiment for example, is available, the presented tool can be used to derive imbibition capillary pressure curves as well as the mentioned parameters.

The use of this technique leads to accurate results for the parameters and the curve fitting purpose. The tool has been designed based on artificial data and shall as a follow up be tested on real centrifuge data and used to determine capillary pressure curves for two-phase flow in mixed-wet reservoirs.

It has to be considered that Excel runs iterations to find the minimum error between two data sets and adjusts the values in the way the minimum error is reached and some deviation in the results from the calculated test data is expected. Nevertheless, it can be observed from the results that the tool works with high accuracy for curve fitting purpose. It was already mentioned in the result section that the curve fitting and adjusting for the parameters is done for each speed independently. As a follow up, a method which accounts for all different centrifuge speeds and iterates to find the “best fit” for all parameters based on available data and not only for one speed is recommended. In the proposed tool, the average value is calculated and is found as a good match.

This tool constitutes a base program to model imbibition capillary pressure curves and the possible improvements to automate this tool have been discussed. This tool is only applicable in two-phase flow and to model three phase flow, it can be extended as the research on three-phase capillary pressure correlations establishes reasonable models.

7. Outlook for three-phase capillary pressure correlation

As highlighted in the introduction, most reservoirs contain three phases but the majority of capillary pressure evaluation techniques have been developed for two phases due to high complexity by adding a third phase. Three-phase capillary pressure curves used to be predicted from two-phase capillary pressure curves which is demonstrated to not work. In three-phase flow, the fluid distribution as well as the displacement process at the pore scale is more difficult than for two-phases. It has to be considered that the dynamics of two different transition zones (oil-gas and oil-water) have to be modelled. Currently, the impact from one movement of a transition zone to the other one is not known. There are infinite possibilities of displacement paths of two independent saturations which makes it really difficult to predict the movement of the flow (Helland and Skjæveland 2004).

Three-phase capillary pressure correlation is a complex research field which attracts attention. Publications about this topic have been made and research is ongoing to find a correlation. Helland and Skjæveland published a suggestion for a three-phase capillary pressure correlation in mixed-wet reservoirs in 2004. They propose a correlation based on Skjæveland et al. (1998) to compute capillary pressure curves for three-phase flow.

In three phase flow, infinite possible ways are available to relate the three phases and therefore an infinite number of unique saturation trajectories exist. Helland and Skjæveland (2004) assume a process-based approach which implements that the processes are known for the saturation trajectories. The saturation change is structured into “classes of processes”. Depending if the saturation is increasing (I), decreasing (D) or constant (C) the phases are classified. Further classification notation, a three-letter symbol is one class. The first letter denotes the direction of water saturation, the second of oil saturation and the third one for gas saturation change. The formulation for the three-phase correlation for each process class is done with the sum of the Brooks-Corey terms for oil and water saturation. This leads to the following equations for all classes of processes.

For the process classes XDI and XID:

$$P_{cgo} = c_g\left(1 - S_g\right)^{-a_g} + c_o(1 - S_o)^{-a_o}. \tag{17}$$

For the process classes IXD and DXI:

$$P_{cgw} = c_g\left(1 - S_g\right)^{-a_g} + c_w(1 - S_w)^{-a_w}. \tag{18}$$

For the process classes DIX and IDX:

$$P_{cow} = c_o(1 - S_o)^{-a_o} + c_w(1 - S_w)^{-a_w}. \tag{19}$$

The X can either stand for I, D or C in all six terms.

It is a reasonable method to start, still the saturation paths are not controlled and the particular direction of the pathway is unknown. It is recommended to combine this method with the one of Yuan and Pope (2012).

Yuan and Popes (2012) approach is independent on phase identification, Gibbs Free Energy (G°) is used to control the system. The minimum G° is reached at a local equilibrium at fixed mole number, pressure and temperature. Using G° the direction of the pathway can be predicted. Finding the minimum G° for a system is difficult and Yuan and Pope (2012) assume to measure relative permeability at known thermodynamic states and interpolate with molar G° of each phase between the states.

Measuring residuals saturations is much more difficult for three phases than in two phases. A lot of different scenarios could occur, for example, all three phases saturations increase, all decrease, one increase - two decrease, two increase - one decrease, one constant - two change and many more.

Yuan and Pope (2012) propose a capillary pressure model consonant with the relative permeability model as well as it should be a continuous function. All parameters of the correlation should be a function of G° of each phase which leads to independency of phase label.

To implement this method in three-phase reservoirs further research is necessary. There are still many unknowns like how to obtain G° for the phases, how to combine them, how to interpret the values to receive the pathway. There is an ongoing research project to use G° in combination with a correlation for capillary pressure curves at the University of Austin in Texas. The aim is to find a way of using G° to determine imbibition capillary pressure curves including hysteresis in three-phase flow.

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Appendix A.

A.1 Example 1

Example 1

```
> restart:
> with(plots):
```

General Definitions for drainage and imbibition capillary pressure curves

```
> Swn:=(S-Swr)/(1-Swr):
  Son:=(1-S-Sor)/(1-Sor):
> pcpi:= cwi/Swn^awi:   pcmi:= coi/Son^aoi:
  pcpd:= cwd/Swn^awd:   pcmd:= cod/Son^aod:
> Pci:= unapply(pcpi+pcmi,S):
  Pcd:= unapply(pcpd+pcmd,S):
  PcD1:= unapply(pcpd,S):
```

Nomenclature:
Swn ... normalized water saturation, Son...normalized oil saturation
pcpi,pcpd ... capillary pressrue positive part of imbibition/drainage
pcmi,pcmd ... capillary pressrue minus part of imbibition/drainage
Pci ... capillary pressure imbibtion, Pcd...capillary pressure drainage
PcD1 ... capillary pressure of primary drainage
cwi,cwd ... entry pressure for water (i for imbibition entry pre and d for drainage entry pre)
coi,cod ... entry pressure for oil (i for imbibition entry pre and d for drainage entry pre)
1/a ... poresize distribution (w...water,o...oil)

Determine coi and cod

```
> coi:=solve(Pci(S1)=0,coi):
  cod:=solve(Pcd(S2)=0,cod):
```

S1 and S2 is the crossover points between respectively primary imbibition and secondary drainage and the "S-axis". Pc(S) is of course zero.

Lands Equation

```
> C:=1/Sor-1/(1-Swr):
```

C...Land's trapping constant
Using Land's equation to determine asymptotes when imbibition starts from primary drainage curve .

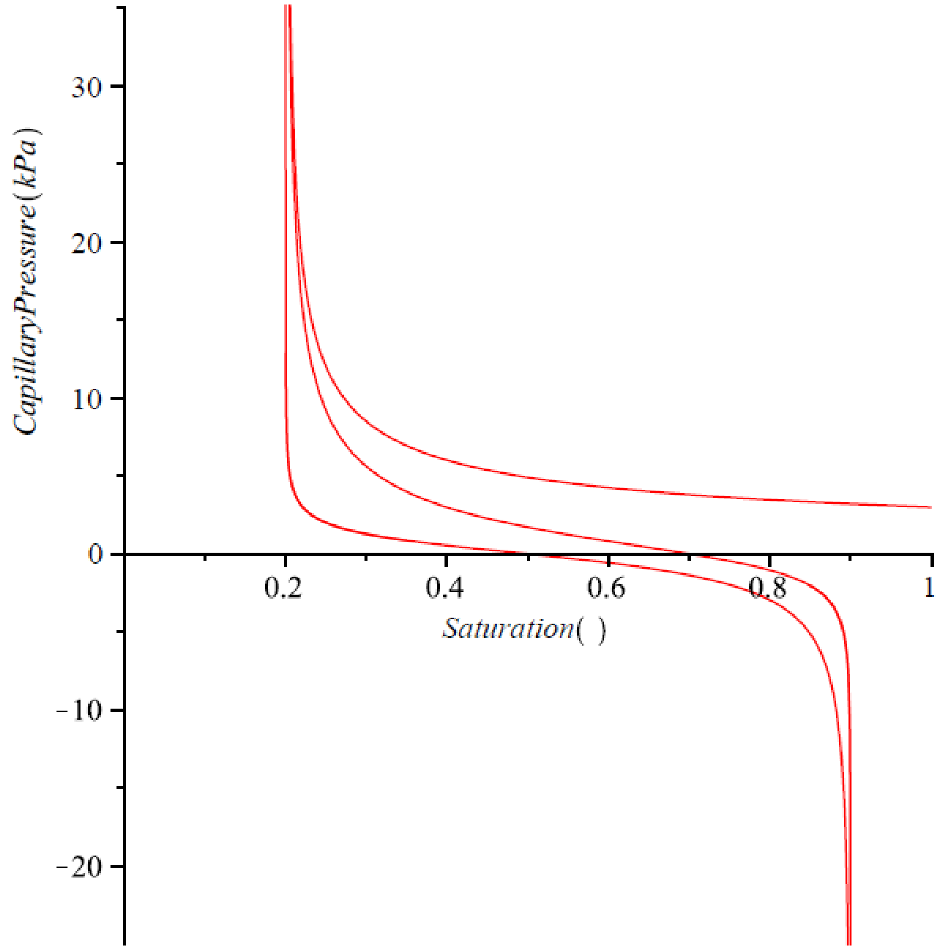
Defining Constants

```
> awi:=0.25: aoi:=0.5: cwi:=2:
  awd:=0.5: aod:=0.25: cwd:=3:
  Sor:=0.1: Swr:=0.2:
  S1:=0.5: S2:=0.7:
```

Plot of primary drainage and imbibiton bounding loop

```
> fpci:= plot(Pci,(Swr-0.05)..(Swr+0.15),0..35,numpoints=500,
  labels=[Saturation ( ),CapillaryPressure (kPa)],thickness=1,
  labeldirections=[horizontal,vertical]):
  fpci1:= plot(Pci,Swr..1,-25..25,numpoints=100,labels=[Saturation
  ( ),CapillaryPressure (kPa)],thickness=1,labeldirections=
  [horizontal,vertical]):
  fpcd:= plot(Pcd, Swr..1,-25..25,numpoints=100,labels=[Saturation
  ( ),CapillaryPressure (kPa)],thickness=1,labeldirections=
  [horizontal,vertical]):
  fpcd1:= plot(Pcd, (1-Sor-0.05)..(1-Sor+0.05),-25..25,numpoints=
  500,labels=[Saturation ( ),CapillaryPressure (kPa)],thickness=1,
  labeldirections=[horizontal,vertical]):
  fpcD1:= plot(PcD1, 0..1,labels=[Saturation ( ),CapillaryPressure
  (kPa)],thickness=1,labeldirections=[horizontal,vertical]):
```

```
> display(fpci1,fpci,fpcd,fpcd1,fpcD1);
```



Reversion

1. Reversal

The first reversal will be an imbibition curve (Pcim[1]) which has its origin on the primary drainage curve (PcD1). The reversal saturation (Srev[1]) is a point on both PcD1 and Pcim[1]. Therefore it is solved for: PcD1(Srev[1])=Pcim[1](Srev[1])

```
> Srev[1]:=0.22:
> SorD[1]:=1/(C+(1/(1-Srev[1]))):
```

This is the residual oil saturation if the spontanous imbibition occurs on the primary drainage curve at the saturtation S=Srev[1]. This is the lowest water saturation experienced.

Calulate the normalized saturations for the first reveral

S_nw ... normalised water saturation

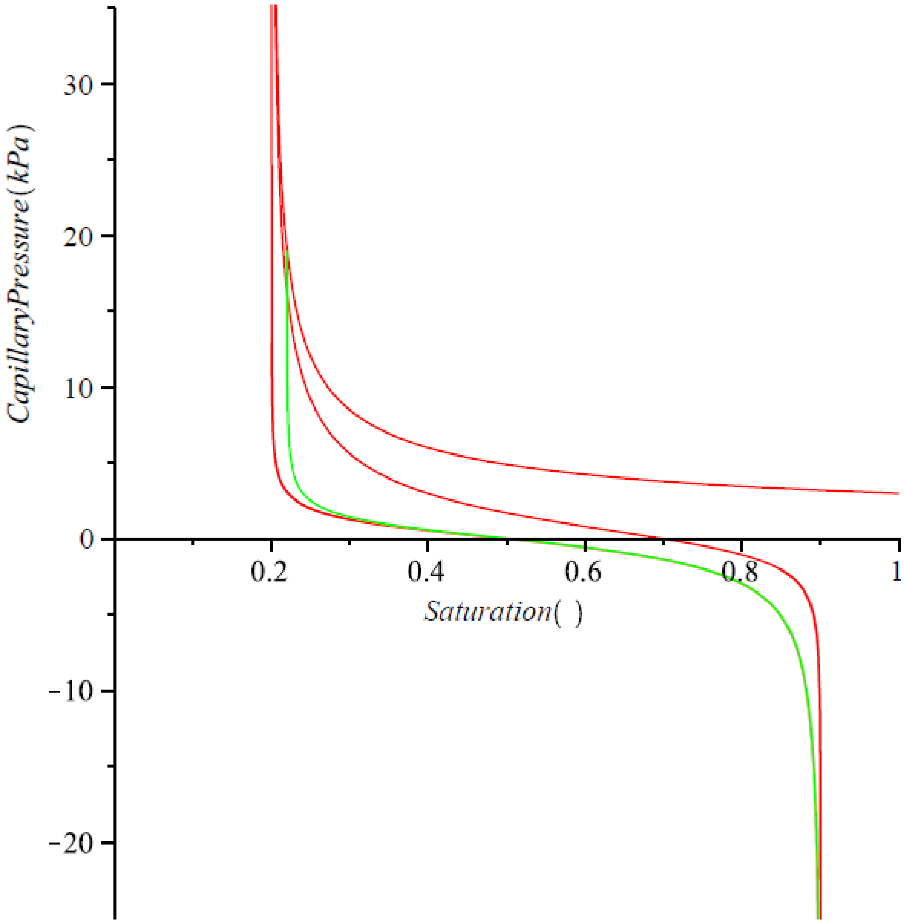
Sw_min1[1] ... asymptote for the positive branch of the imbibition curve

S_no ... normalised water saturation for the negative branch

SorD[1] ... asymptote for the negative branch

```
> S_nw[1]:=(S-Sw_min1[1])/(1-Sw_min1[1]):
> S_no[1]:=(1-S-SorD[1])/(1-SorD[1]):
> Pcim[1]:=cwi/S_nw[1]^awi+coi/S_no[1]^aoi:
  Pcim[1]:=unapply(Pcim[1],S):
> Sw_min1[1]:=solve(Pcim[1](Srev[1])=PcD1(Srev[1]),Sw_min1[1]):
```

```
> fpcirl:=plot(Pcim[1],Srev[1]..0.99,-25..10,numpoints=300,color=
  green,labels=[Saturation ( ),CapillaryPressure (kPa)],thickness=
  1,labeldirections=[horizontal,vertical]):
> display(fpci1,fpci,fpcd,fpcd1,fpcD1,fpcirl);
```



2.Reversal

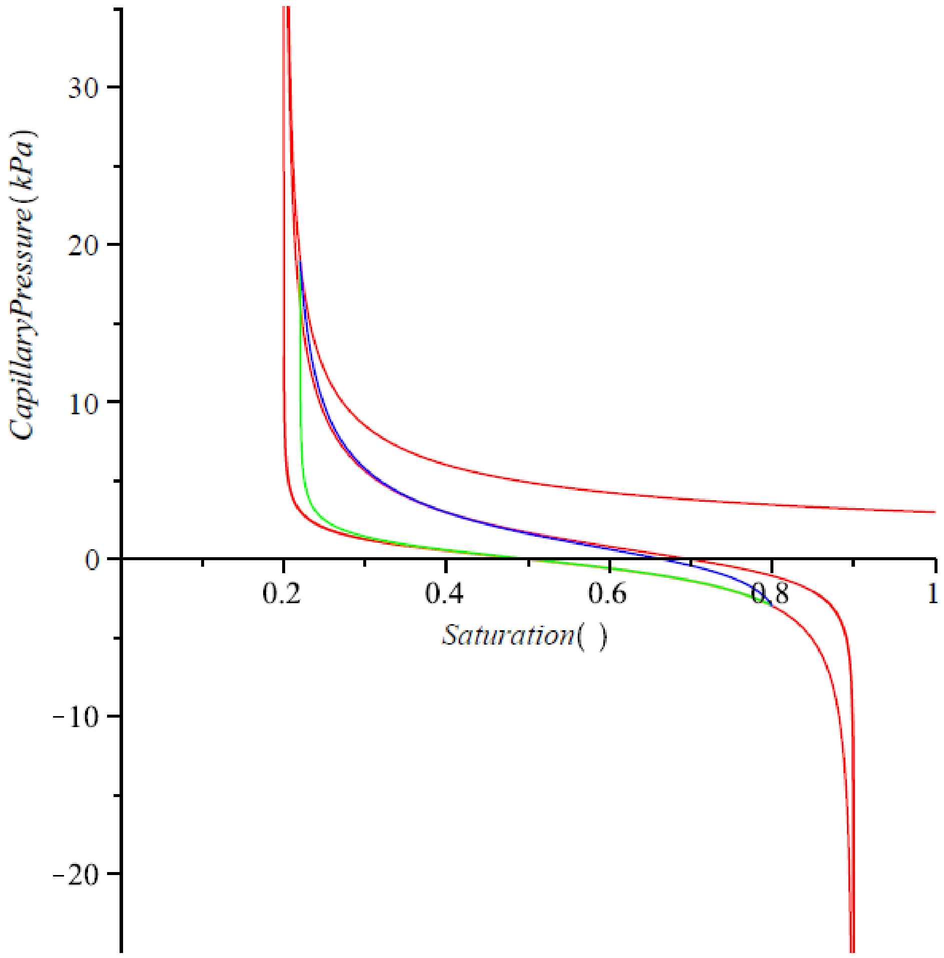
the secound reversal starts at the point Srev[2]. the secound reversal is a drainage cuvre starting on the imbibition curve from the first reversal and ending on the primary drainage curve (PcD1), the drainage curve has to scann back to to Srev[1] to close the loop.
First we have to assume that Sw_min2=Sw_min1 to start the process, as we have two unkowns this time, Sw_min2 and So_min2. We have 2 equations which we have to flip-flop until they converge.
epsw,epso,b,a are varibales which help to store results to test the loop on convergence
The two equations to be solved for are:
Pcim[1](Srev[2])=Pcdr[2](Srev[2])
Pcim[1](Srev[1])=Pcdr[2](Srev[1])

```
> Srev[2]:=0.8:
> Sw_min2[1]:=Sw_min1[1]:
> epsw[1]:=2:
  epso[1]:=2:
  b[0]:=1:
  a[0]:=1:
  for i from 1 to 10 do
```

```
S_nw:=(S-Sw_min2[i])/(1-Sw_min2[i]):
S_no:= (1-S-So_min2[i])/(1-So_min2[i]):
Pcdr[2]:= cwd/S_nw^awd+cod/S_no^aod:
Pcdr[2]:= unapply(Pcdr[2],S):
So_min2[i]:=solve(Pcim[1](Srev[2])=Pcdr[2](Srev[2]),So_min2[i]):
a[i]:=So_min2[i]:
S_nw:= (S-Sw_min2[i+1])/(1-Sw_min2[i+1]):
S_no:= (1-S-So_min2[i])/(1-So_min2[i]):
Pcdr[2]:= cwd/S_nw^awd+cod/S_no^aod:
Pcdr[2]:= unapply(Pcdr[2],S):
Sw_min2[i+1]:= solve(Pcim[1](Srev[1])=Pcdr[2](Srev[1]),Sw_min2
[i+1]):
b[i]:=Sw_min2[i+1]:
epso[i]:=abs(a[i]-a[i-1]):
epsw[i]:=abs(b[i]-b[i-1]):
eps[i]:=epsw[i]+epso[i]:
print(eps[i]);
if (eps[i]<0.002)
then
print(i);
i:=11
fi:
od:
```

```
                                1.617240443
                                0.0001604736
                                     2                                  (1)
```

```
> Sw_min2[3]:
> fpcdr[2]:=plot(Pcdr[2],Srev[1]..Srev[2], color=blue,labels=
  [Saturation ( ),CapillaryPressure (kPa)],thickness=1,
  labeldirections=[horizontal,vertical]):
> fpcim[1]:=plot(Pcim[1],Srev[1]..Srev[2], color=green,labels=
  [Saturation ( ),CapillaryPressure (kPa)],thickness=1,
  labeldirections=[horizontal,vertical]):
> display(fpci1,fpci,fpcd,fpcd1,fpcD1,fpcim[1],fpcdr[2]);
```



3. Reversal

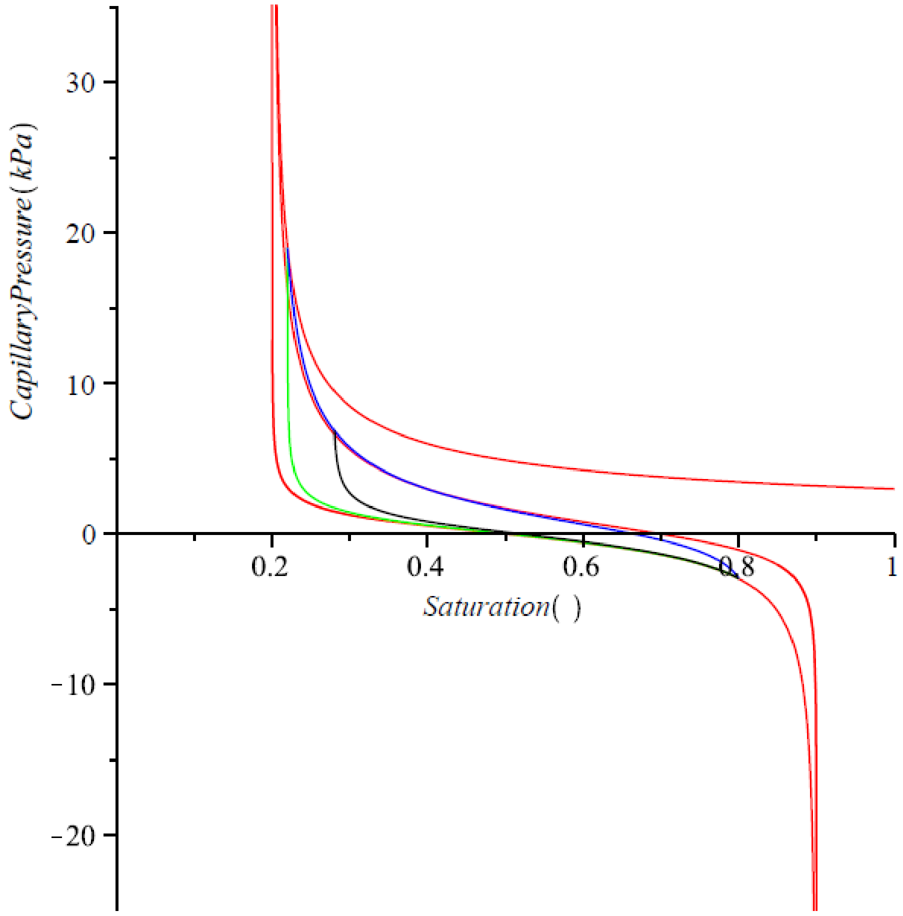
The third reversal starts on the Pcdr[2] curve before the Srev[1] point is reached and is an imbibition curve. The imbibiton curve starts at Srev[3] .
The two equations to be solved for are:
Pcim[3](Srev[2])=Pcdr[2](Srev[2])
Pcim[3](Srev[3])=Pcdr[2](Srev[3])

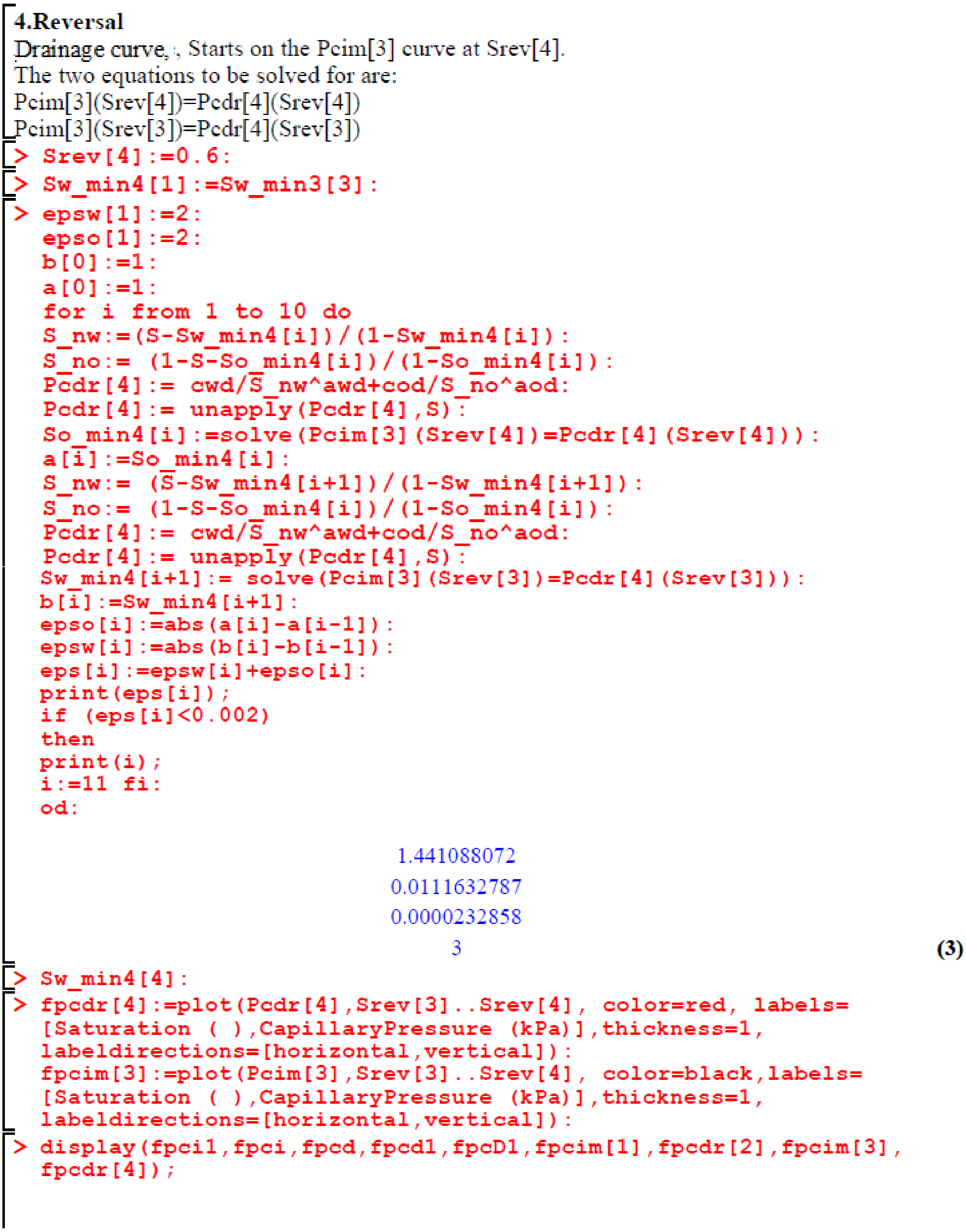
```
> Srev[3]:=0.28:
> Sw_min3[1]:=Sw_min2[3]:
> epsw[1]:=2:
  epso[1]:=2:
  b[0]:=1:
  a[0]:=1:
  for i from 1 to 10 do
  S_nw:=(S-Sw_min3[i])/(1-Sw_min3[i]):
  S_no:= (1-S-So_min3[i])/(1-So_min3[i]):
  Pcim[3]:= cwi/S_nw^awi+coi/S_no^aoi:
  Pcim[3]:= unapply(Pcim[3],S):
  So_min3[i]:=solve(Pcim[3](Srev[2])=Pcdr[2](Srev[2]),So_min3[i]):
  a[i]:=So_min3[i]:
  S_nw:= (S-Sw_min3[i+1])/(1-Sw_min3[i+1]):
  S_no:= (1-S-So_min3[i])/(1-So_min3[i]):
```

```
Pcim[3]:= cwi/S_nw^awi+coi/S_no^aoi:
Pcim[3]:= unapply(Pcim[3],S):
Sw_min3[i+1]:= solve(Pcim[3](Srev[3])=Pcdr[2](Srev[3]),Sw_min3
[i+1]):
b[i]:=Sw_min3[i+1]:
epso[i]:=abs(a[i]-a[i-1]):
epsw[i]:=abs(b[i]-b[i-1]):
eps[i]:=epsw[i]+epso[i]:
print(eps[i]);
if (eps[i]<0.002)
then
print(i);
i:=11;
fi:
od:
```

```
                              1.622294593
                             0.00082583161
                                   2                                (2)
```

```
> Sw_min3[3]:
> fpcim[3]:=plot(Pcim[3],Srev[3]..Srev[2], color=black,labels=
  [Saturation ( ),CapillaryPressure (kPa)],thickness=1,
  labeldirections=[horizontal,vertical]):
> display(fpci1,fpci,fpcd,fpcd1,fpcD1,fpcim[1],fpcdr[2],fpcim[3]);
```





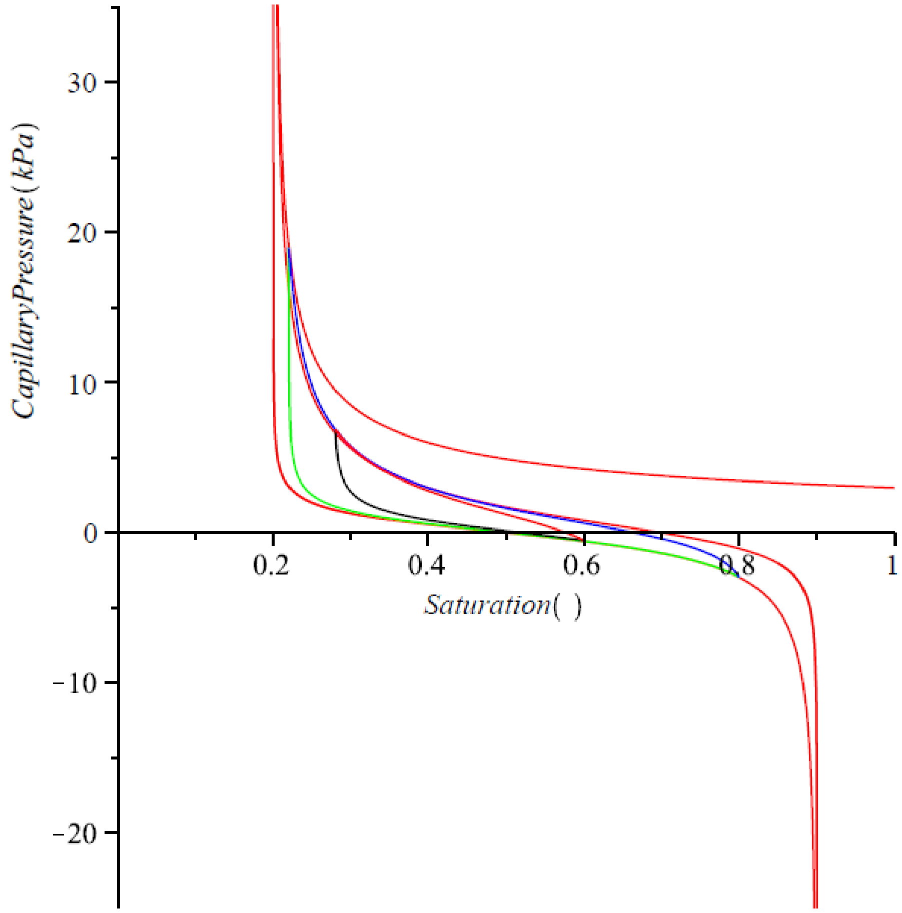
4.Reversal

Drainage curve, Starts on the Pcim[3] curve at Srev[4].
The two equations to be solved for are:
Pcim[3](Srev[4])=Pcdr[4](Srev[4])
Pcim[3](Srev[3])=Pcdr[4](Srev[3])

```
> Srev[4]:=0.6:
> Sw_min4[1]:=Sw_min3[3]:
> epsw[1]:=2:
  epso[1]:=2:
  b[0]:=1:
  a[0]:=1:
  for i from 1 to 10 do
  S_nw:=(S-Sw_min4[i])/(1-Sw_min4[i]):
  S_no:= (1-S-So_min4[i])/(1-So_min4[i]):
  Pcdr[4]:= cwd/S_nw^awd+cod/S_no^aod:
  Pcdr[4]:= unapply(Pcdr[4],S):
  So_min4[i]:=solve(Pcim[3](Srev[4])=Pcdr[4](Srev[4])):
  a[i]:=So_min4[i]:
  S_nw:= (S-Sw_min4[i+1])/(1-Sw_min4[i+1]):
  S_no:= (1-S-So_min4[i])/(1-So_min4[i]):
  Pcdr[4]:= cwd/S_nw^awd+cod/S_no^aod:
  Pcdr[4]:= unapply(Pcdr[4],S):
  Sw_min4[i+1]:= solve(Pcim[3](Srev[3])=Pcdr[4](Srev[3])):
  b[i]:=Sw_min4[i+1]:
  epso[i]:=abs(a[i]-a[i-1]):
  epsw[i]:=abs(b[i]-b[i-1]):
  eps[i]:=epsw[i]+epso[i]:
  print(eps[i]);
  if (eps[i]<0.002)
  then
  print(i);
  i:=11 fi:
  od:
```

```
                              1.441088072
                             0.0111632787
                             0.0000232858
                                   3                              (3)
```

```
> Sw_min4[4]:
> fpcdr[4]:=plot(Pcdr[4],Srev[3]..Srev[4], color=red, labels=
  [Saturation ( ),CapillaryPressure (kPa)],thickness=1,
  labeldirections=[horizontal,vertical]):
  fpcim[3]:=plot(Pcim[3],Srev[3]..Srev[4], color=black,labels=
  [Saturation ( ),CapillaryPressure (kPa)],thickness=1,
  labeldirections=[horizontal,vertical]):
> display(fpci1,fpci,fpcd,fpcd1,fpcD1,fpcim[1],fpcdr[2],fpcim[3],
  fpcdr[4]);
```



```
> display(fpci1,fpci,fpcd,fpcd1,fpcD1,fpcim[1],fpcdr[2],fpcim[3],
  fpcdr[4], view=[0.25..0.65,-10..15]):
```

Reversal 5

The fifth reversal starts on the Pcdr[4] curve before the Srev[3] point is reached and is an imbibition curve. The imbibiton curve starts at Srev[3] .

The two equations to be solved for are:

Pcim[5](Srev[4])=Pcdr[4](Srev[4])

Pcim[5](Srev[5])=Pcdr[4](Srev[5])

```
> Srev[5]:=0.35:
```

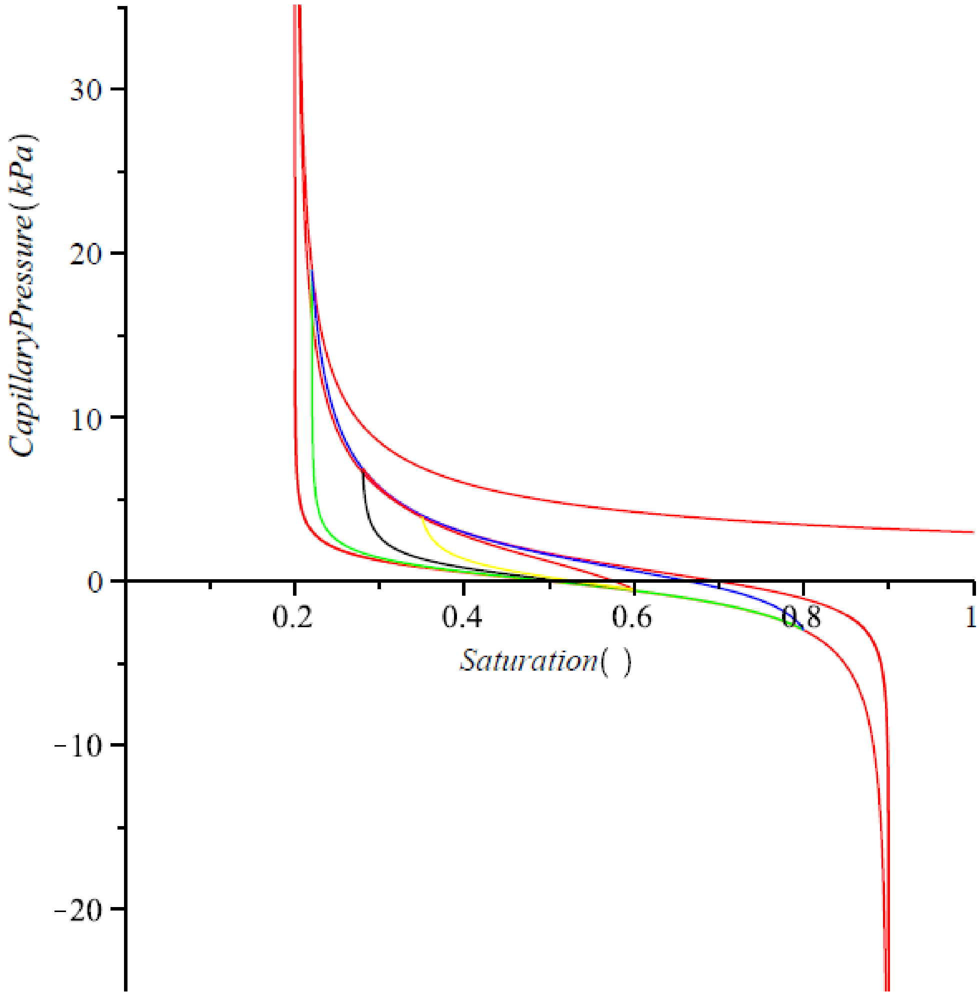
```
> Sw_min5[1]:=Sw_min4[4]:
```

```
> epsw[1]:=2:
  epso[1]:=2:
  b[0]:=1:
  a[0]:=1:
  for i from 1 to 10 do
  S_nw:=(S-Sw_min5[i])/(1-Sw_min5[i]):
  S_no:= (1-S-So_min5[i])/(1-So_min5[i]):
  Pcim[5]:= cwi/S_nw^awi+coi/S_no^aoi:
  Pcim[5]:= unapply(Pcim[5],S):
  So_min5[i]:=solve(Pcim[5](Srev[4])=Pcdr[4](Srev[4]),So_min5[i]):
  a[i]:=So_min5[i]:
  S_nw:= (S-Sw_min5[i+1])/(1-Sw_min5[i+1]):
```

```
S_no:= (1-S-So_min5[i])/(1-So_min5[i]):
Pcim[5]:= cwi/S_nw^awi+coi/S_no^aoi:
Pcim[5]:= unapply(Pcim[5],S):
Sw_min5[i+1]:= solve(Pcim[5](Srev[5])=Pcdr[4](Srev[5]),Sw_min5
[i+1]):
b[i]:=Sw_min5[i+1]:
epso[i]:=abs(a[i]-a[i-1]):
epsw[i]:=abs(b[i]-b[i-1]):
eps[i]:=epsw[i]+epso[i]:
print(eps[i]);
if (eps[i]<0.002)
then
print(i);
i:=11 fi:
od:
```

```
                        1.578037657
                       0.04369505698
                       0.0000685697
                             3                              (4)
```

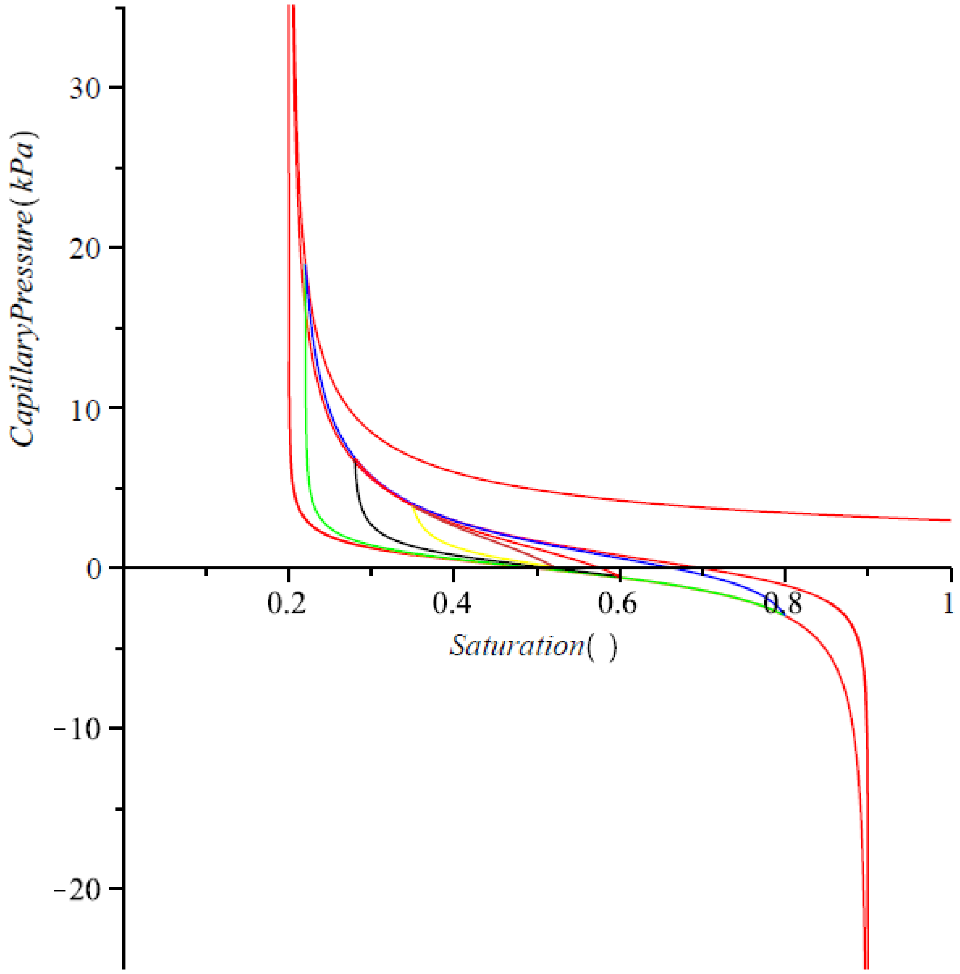
```
> fpcim[5]:=plot(Pcim[5],Srev[5]..Srev[4], color=yellow,labels=
  [Saturation ( ),CapillaryPressure (kPa)],thickness=1,
  labeldirections=[horizontal,vertical]):
> display(fpci1,fpci,fpcd,fpcd1,fpcD1,fpcim[1],fpcdr[2],fpcim[3],
  fpcdr[4],fpcim[5]);
```



Reversal 6

Drainage curve,, Starts on the Pcim[5] curve at Srev[6]
The two equations to be solved for are:
Pcim[5](Srev[6])=Pcdr[6](Srev[6])
Pcim[5](Srev[5])=Pcdr[6](Srev[5])

```
> Srev[6]:=0.52:
> Sw_min6[1]:=Sw_min5[4]:
> epsw[1]:=2:
  epso[1]:=2:
  b[0]:=1:
  a[0]:=1:
  for i from 1 to 10 do
  S_nw:=(S-Sw_min6[i])/(1-Sw_min6[i]):
  S_no:= (1-S-So_min6[i])/(1-So_min6[i]):
  Pcdr[6]:= cwd/S_nw^awd+cod/S_no^aod:
  Pcdr[6]:= unapply(Pcdr[6],S):
  So_min6[i]:=solve(Pcim[5](Srev[6])=Pcdr[6](Srev[6])):
  a[i]:=So_min4[i]:
  S_nw:= (S-Sw_min6[i+1])/(1-Sw_min6[i+1]):
  S_no:= (1-S-So_min6[i])/(1-So_min6[i]):
  Pcdr[6]:= cwd/S_nw^awd+cod/S_no^aod:
  Pcdr[6]:= unapply(Pcdr[6],S):
  Sw_min6[i+1]:= solve(Pcim[5](Srev[5])=Pcdr[6](Srev[5])):
  b[i]:=Sw_min6[i+1]:
  epso[i]:=abs(a[i]-a[i-1]):
  epsw[i]:=abs(b[i]-b[i-1]):
  eps[i]:=epsw[i]+epso[i]:
  print(eps[i]);
  if (eps[i]<0.002)
  then
  print(i);
  i:=11 fi:
  od:
                                   1.430863836
                                  0.0142699025
                                  0.0001045167
                                        3                                  (5)
> fpcdr[6]:=plot(Pcdr[6],Srev[5]..Srev[6], color=orange,labels=
  [Saturation ( ),CapillaryPressure (kPa)],thickness=1,
  labeldirections=[horizontal,vertical]):
  fpcim[5]:=plot(Pcim[5],Srev[5]..Srev[6], color=yellow,labels=
  [Saturation ( ),CapillaryPressure (kPa)],thickness=1,
  labeldirections=[horizontal,vertical]):
> display(fpci1,fpci,fpcd,fpcd1,fpcD1,fpcim[1],fpcdr[2],fpcim[3],
  fpcdr[4],fpcim[5],fpcdr[6]);
```



```
> display(fpci1,fpci,fpcd,fpcd1,fpcD1,fpcim[1],fpcdr[2],fpcim[3],
  fpcdr[4],fpcim[5],fpcdr[6], view=[0.18..0.82,-10..20]):
>
```

A.2 Example 2

```
> restart:
```

Example 2

```
> with(plots):
> with(RealDomain):
```

General Definitions for drainage and imbibition capillary pressure curves

```
> Swn:=(S-Swr)/(1-Swr):
  Son:=(1-S-Sor)/(1-Sor):
> pcpi:= cwi/Swn^awi:  pcmi:= coi/Son^aoi:
  pcpd:= cwd/Swn^awd:  pcmd:= cod/Son^aod:
> Pci:= unapply(pcpi+pcmi,S):
  Pcd:= unapply(pcpd+pcmd,S):
  PcD1:= unapply(pcpd,S):
```

Swn ... normalized water saturation. Son ... normalized oil saturation
pcpi,pcpd ...c apillary pressrue positive part of imbibition/drainage
pcmi,pcmd ... capillary pressrue minus part of imbibition/drainage
Pci ... capillary pressure imbibtion, Pcd ... capillary pressure drainage
PcD1 ... capillary pressure of primary drainage
cwi,cwd ... entry pressure for water (i for imbibition entry pre and d for drainage entry pre)
coi,cod ... entry pressure for oil (i for imbibition entry pre and d for drainage entry pre)
1/a .. .poresize distribution (w...water,o...oil)

Determine coi and cod

```
> coi:=solve(Pci(S1)=0,coi):
  cod:=solve(Pcd(S2)=0,cod):
```

S1 and S2 is the crossover points between respectively primary imbibition and secondary drainage and the "S-axis". Pc(S) is of course zero.

Lands Equation

```
> C:=1/Sor-1/(1-Swr):
```

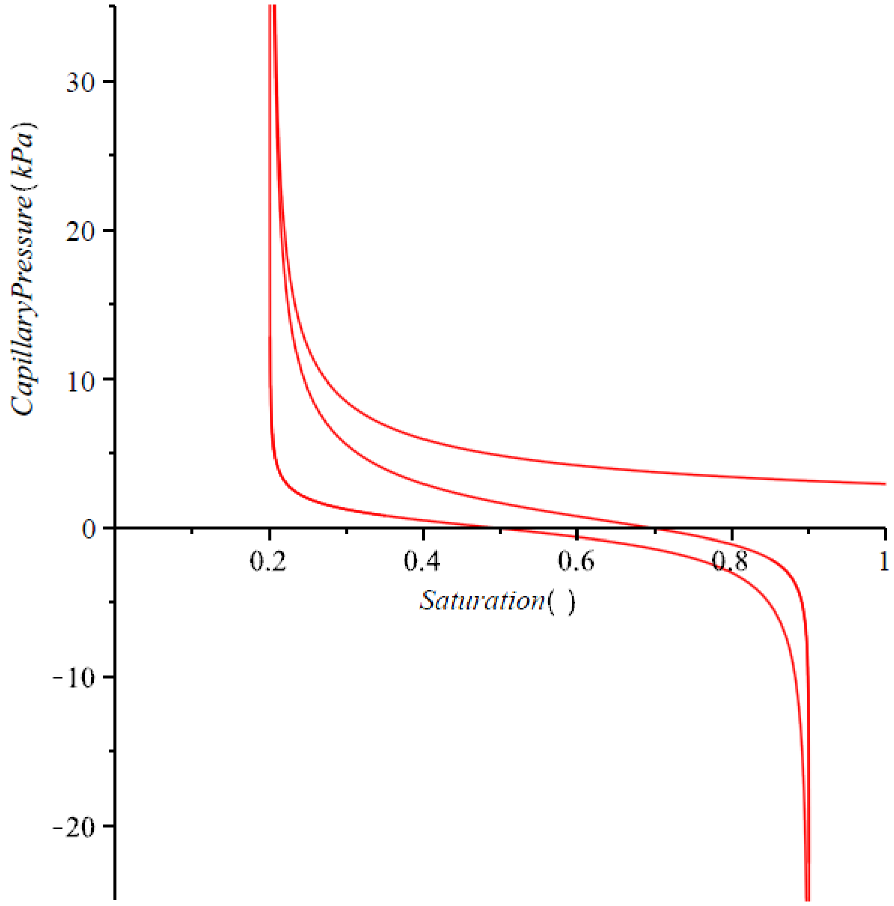
C ... Land's trapping constant
Using Land's equation to determine asymptotes when imbibition starts from primary drainage curve .

Defining Constants

```
> awi:=0.25: aoi:=0.5: cwi:=2:
  awd:=0.5: aod:=0.25: cwd:=3:
  Sor:=0.1: Swr:=0.2:
  S1:=0.5: S2:=0.7:
```

Plot of primary drainage and imbibiton bounding loop

```
> fpci:= plot(Pci,(Swr-0.05)..(Swr+0.15),0..35,numpoints=
  500,labels=[Saturation ( ),CapillaryPressure (kPa)],thickness=1,
  labeldirections=[horizontal,vertical]):
  fpci1:= plot(Pci,Swr..1,-25..25,numpoints=100,labels=[Saturation
  ( ),CapillaryPressure (kPa)],thickness=1,labeldirections=
  [horizontal,vertical]):
  fpcd:= plot(Pcd, Swr..1,-25..25,numpoints=100,labels=[Saturation
  ( ),CapillaryPressure (kPa)],thickness=1,labeldirections=
  [horizontal,vertical]):
  fpcd1:= plot(Pcd, (1-Sor-0.05)..(1-Sor+0.05),-25..25,numpoints=
  500,labels=[Saturation ( ),CapillaryPressure (kPa)],thickness=1,
  labeldirections=[horizontal,vertical]):
  fpcD1:= plot(PcD1, 0..1,labels=[Saturation ( ),CapillaryPressure
  (kPa)],thickness=1,labeldirections=[horizontal,vertical]):
> display(fpci1,fpci,fpcd,fpcd1,fpcD1);
```



The first 2 reversals have different datapoints, which was found out by trail and error. The other reversals (3 to 30) it was possible to see a realtionship and general input data was created.

Reversal

First 2 reversal

have different input values then the other reversals:

1. Reversal

```
> Srev[1]:=0.95:
```

```
> SorD[1]:=1/(C+(1/(1-Srev[1]))):
```

This is the residual oil saturation if the process (primary drainage) i reversed from S=Srev[1]. This is the lowest water saturation experienced.

Calulate the normalized saturations for the first reveral:
S_nw ... normalised water saturation
Sw_min1[1] ... asymptote for the positive branch of the imbibition curve
S_no ... normalised water saturation for the negative branch
SorD[1] ... asymptote for the negative branch

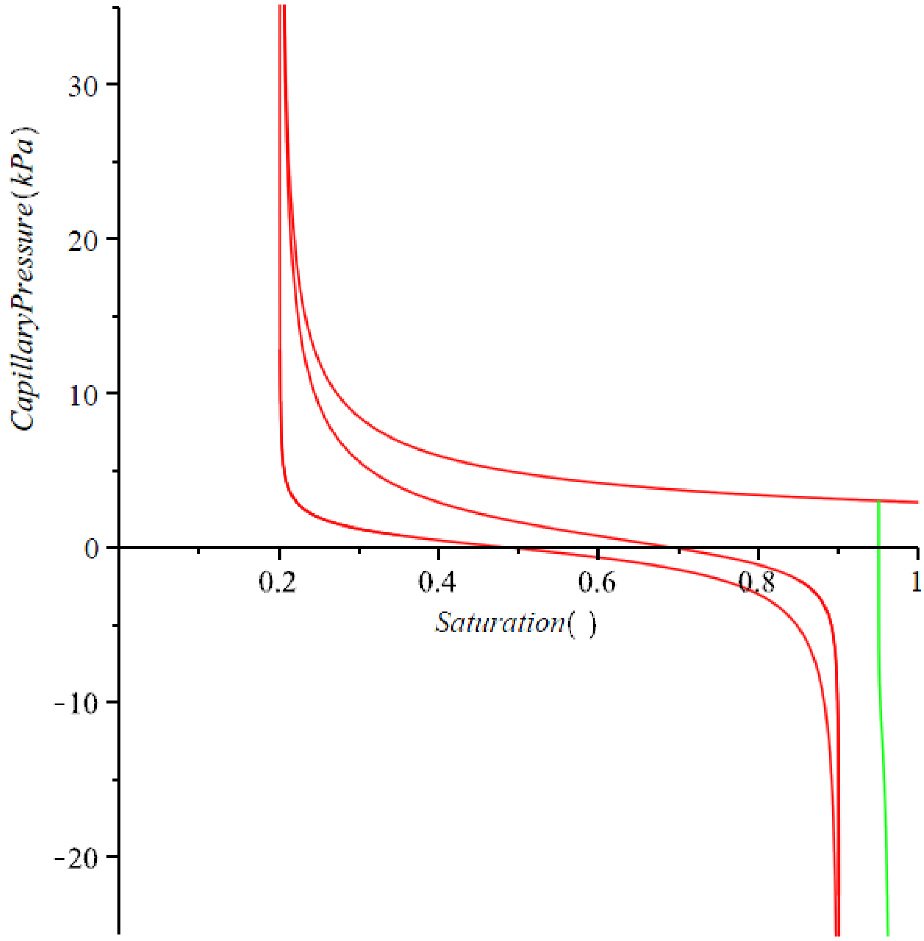
```
> S_nw[1]:=(S-Sw_min1[1])/(1-Sw_min1[1]):
```

```
> S_no[1]:=(1-S-SorD[1])/(1-SorD[1]):
```

```
> Pcim[1]:=cwi/S_nw[1]^awi+coi/S_no[1]^aoi:
  Pcim[1]:=unapply(Pcim[1],S):
```

```
> Sw_min1[1]:=solve(Pcim[1](Srev[1])=PcD1(Srev[1]),Sw_min1[1]):
```

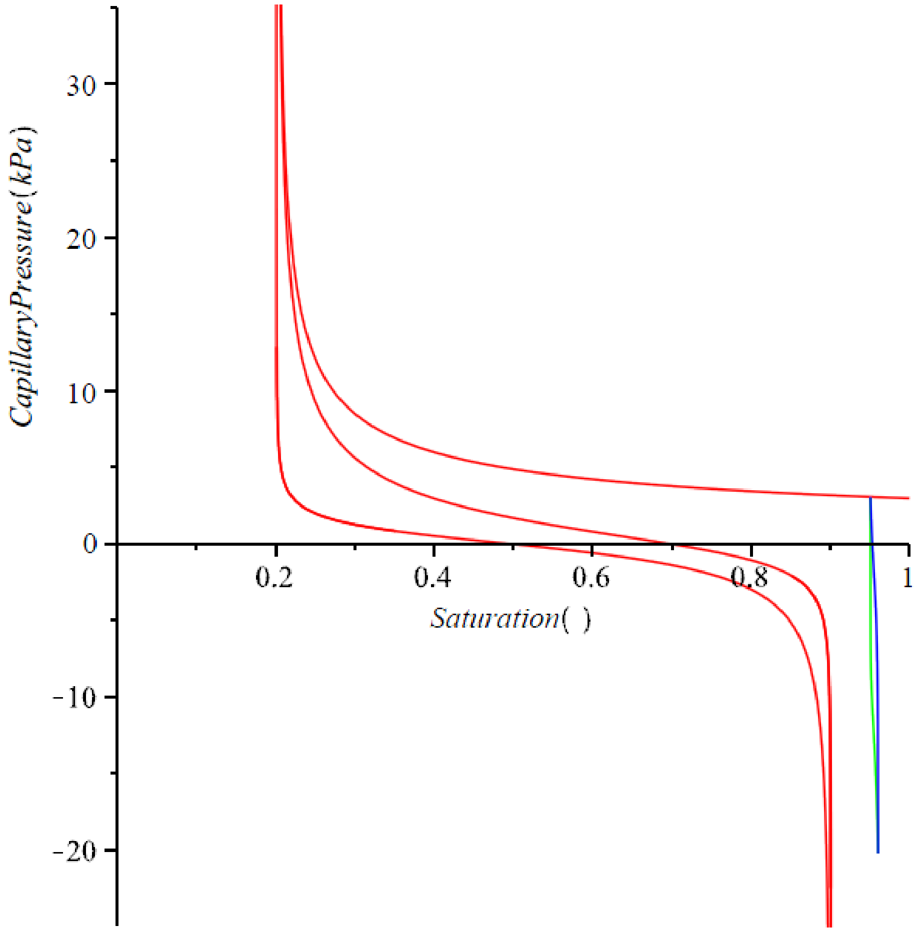
```
> fpcir1:=plot(Pcim[1],Srev[1]..0.99,-25..10,numpoints=300,color=
  green,labels=[Saturation ( ),CapillaryPressure (kPa)],thickness=
  1,labeldirections=[horizontal,vertical]):
> display(fpci1,fpci,fpcd,fpcd1,fpcD1,fpcir1);
```



2.Reversal

```
> Srev[2]:=0.96:
> Sw_min2[1]:=Sw_min1[1]:
> epsw[1]:=2:
  epso[1]:=2:
  b[0]:=1:
  a[0]:=1:
  for i from 1 to 10 do
  S_nw:=(S-Sw_min2[i])/(1-Sw_min2[i]):
  S_no:=  (1-S-So_min2[i])/(1-So_min2[i]):
  Pcdr[2]:= cwd/S_nw^awd+cod/S_no^aod:
  Pcdr[2]:= unapply(Pcdr[2],S):
  So_min2[i]:=solve(Pcim[1](Srev[2])=Pcdr[2](Srev[2]),So_min2[i]):
  a[i]:=So_min2[i]:
  S_nw:= (S-Sw_min2[i+1])/(1-Sw_min2[i+1]):
  S_no:= (1-S-So_min2[i])/(1-So_min2[i]):
  Pcdr[2]:= cwd/S_nw^awd+cod/S_no^aod:
  Pcdr[2]:= unapply(Pcdr[2],S):
```

```
Sw_min2[i+1]:= solve(Pcim[1](Srev[1])=Pcdr[2](Srev[1]),Sw_min2
[i+1]):
b[i]:=Sw_min2[i+1]:
epso[i]:=abs(a[i]-a[i-1]):
epsw[i]:=abs(b[i]-b[i-1]):
eps[i]:=epsw[i]+epso[i]:
#print(eps[i]);
if (eps[i]<0.002)
then
#print(i);
i:=11
fi:
od:
> fpcdr2:=plot(Pcdr[2],Srev[1]..Srev[2], color=blue,labels=
[Saturation ( ),CapillaryPressure (kPa)],thickness=1,
labeldirections=[horizontal,vertical]):
fpcir1:=plot(Pcim[1],Srev[1]..Srev[2],color=green,labels=
[Saturation ( ),CapillaryPressure (kPa)],thickness=1,
labeldirections=[horizontal,vertical]):
> display(fpci1,fpci,fpcd,fpcd1,fpcD1,fpcir1,fpcdr2);
```



Datapoints

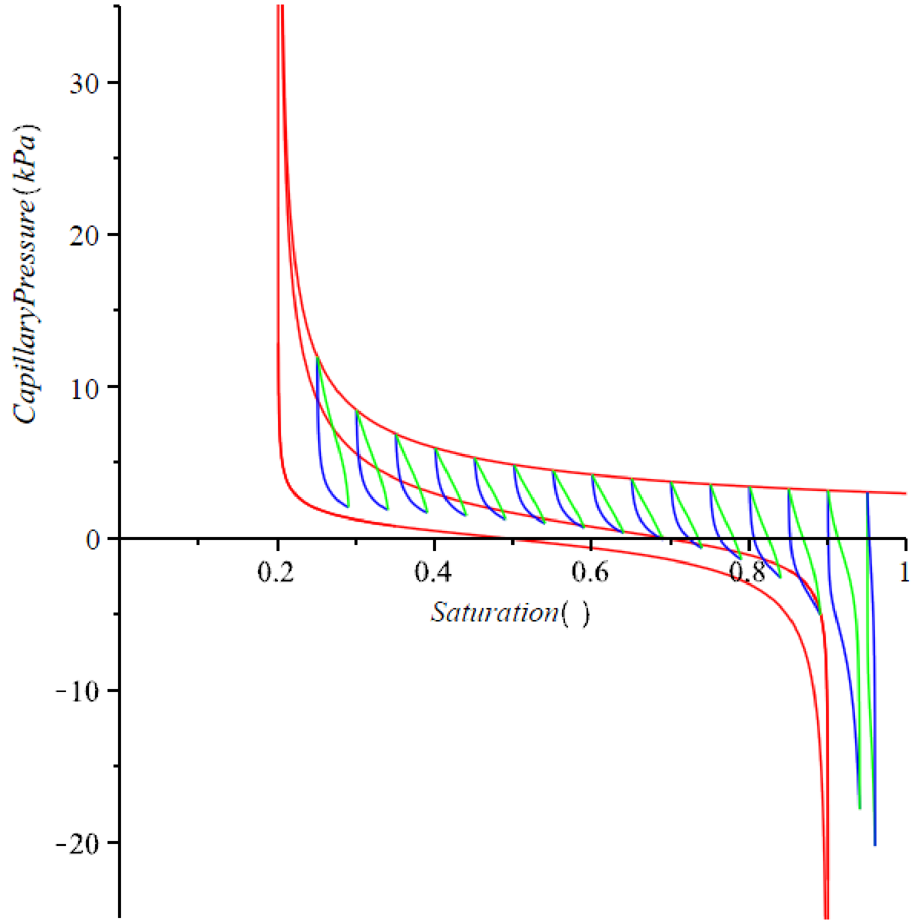
```
> k:=0.1:
```

```
> for i from 3 to 38 do
  if type(i,odd) then
  Srev[i]:=1-k;
  k:=k+0.05:
  else
  Srev[i]:= Srev[i-3]-0.01:
  fi:
  od:
> #for i from 3 to 38 do
  #print(Srev = Srev[i]):
  #od:
```

From 3th to the 30th reversal

```
> for j from 3 to 30 do
  #print("j" = j);

  #print(Srev = Srev[j]);

  if type(j,odd) then

  Srev[j]:
  SorD[j]:=1/(C+(1/(1-Srev[j]))):
  S_nw[j]:=(S-Sw_mini[j])/(1-Sw_mini[j]):
  S_no[j]:=(1-S-SorD[j])/(1-SorD[j]):
  Pcim[j]:=cwi/S_nw[j]^awi+coi/S_no[j]^aoi:
  Pcim[j]:=unapply(Pcim[j],S);
  Sw_mini[j]:=solve(Pcim[j](Srev[j])=PcD1(Srev[j]),Sw_mini[j]);
  #print(Sw_mini = Sw_mini[j]);
  #print("------------------------------------");
  fpcir[j]:=plot(Pcim[j],Srev[j]..0.99,-25..10,numpoints=300,color=
  blue,labels=[Saturation ( ),CapillaryPressure (kPa)],thickness=1,
  labeldirections=[horizontal,vertical]):

  else

  #print(Srev = Srev[j]);
  epsw[1]:=2:
  epso[1]:=2:
  b[0]:=1:
  a[0]:=1:
  Sw_min[j,1]:=Sw_mini[j-1];

  for i from 1 to 10 do
  S_nw:=(S-Sw_min[j,i])/(1-Sw_min[j,i]);
  S_no:=(1-S-So_min[j,i])/(1-So_min[j,i]);
  Pcdr[j]:= cwd/S_nw^awd+cod/S_no^aod;
  Pcdr[j]:= unapply(Pcdr[j],S);
  So_min[j,i]:=solve(Pcim[j-1](Srev[j])=Pcdr[j](Srev[j]),So_min[j,
  i]);
  a[i]:=So_min[j,i];
  S_nw:= (S-Sw_min[j,i+1])/(1-Sw_min[j,i+1]);
  S_no:= (1-S-So_min[j,i])/(1-So_min[j,i]);
  Pcdr[j]:= cwd/S_nw^awd+cod/S_no^aod;
  Pcdr[j]:= unapply(Pcdr[j],S);
  Sw_min[j,i+1]:= solve(Pcim[j-1](Srev[j-1])=Pcdr[j](Srev[j-1]),
  Sw_min[j,i+1]);
  b[i]:=Sw_min[j,i+1];
```

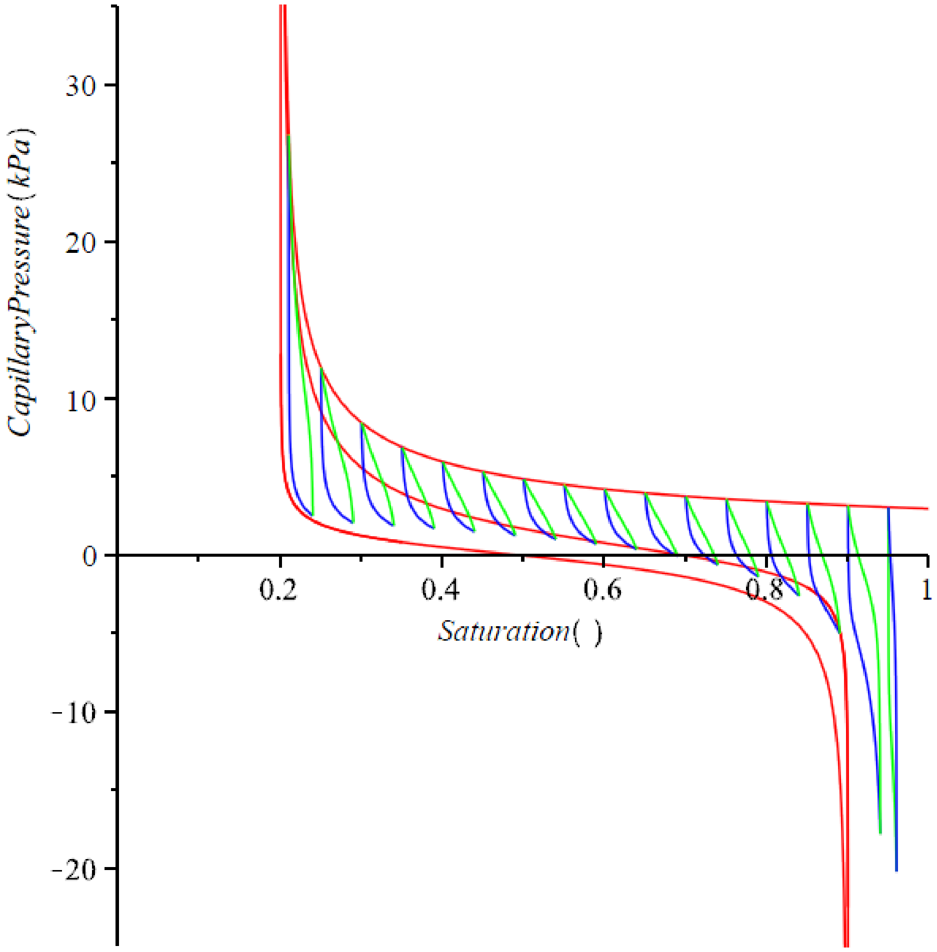
```
epso[i]:=abs(a[i]-a[i-1]);
epsw[i]:=abs(b[i]-b[i-1]);
eps[i]:=epsw[i]+epso[i];
#print(eps = eps[i]);

if (eps[i]<0.002) then
#print("number of iterations:",i);
Sw_min[j]:=Sw_min[j,i];
So_min[j]:=So_min[j,i];
i:=11;
fi:

fpcdr[j]:=plot(Pcdr[j],Srev[j-1]..Srev[j], color=green,labels=
[Saturation ( ),CapillaryPressure (kPa)],thickness=1,
labeldirections=[horizontal,vertical]):
fpcir[j-1]:=plot(Pcim[j-1],Srev[j-1]..Srev[j], color=blue,labels=
[Saturation ( ),CapillaryPressure (kPa)],thickness=1,
labeldirections=[horizontal,vertical]):
od;

#print(Sw_min = Sw_min[j]);
#print(So_min = So_min[j]);
#print("----------------------------------");
fi:
od;
> display(fpci1,fpci,fpcd,fpcd1,fpcD1,fpcir1,fpcdr2,fpcir[3],fpcdr
[4],fpcir[5],fpcdr[6],fpcir[7],fpcdr[8],fpcir[9],fpcdr[10],fpcir
[11],fpcdr[12],fpcir[13],fpcdr[14],fpcir[15],fpcdr[16],fpcir[17],
fpcdr[18],fpcir[19],fpcdr[20],fpcir[21],fpcdr[22],fpcir[23],fpcdr
[24],fpcir[25],fpcdr[26],fpcir[27],fpcdr[28],fpcir[29],fpcdr[30])
;
```



Last possible reversal:

```
> Srev[31]:=0.21:
  SorD[31]:=1/(C+(1/(1-Srev[31]))):
  S_nw[31]:=(S-Sw_mini[31])/(1-Sw_mini[31]):
  S_no[31]:=(1-S-SorD[31])/(1-SorD[31]):
  Pcim[31]:=cwi/S_nw[31]^awi+coi/S_no[31]^aoi:
  Pcim[31]:=unapply(Pcim[31],S):
  Sw_mini[31]:=solve(Pcim[31](Srev[31])=PcD1(Srev[31]),Sw_mini[31])
  :
  #print(Sw_mini = Sw_mini[31]):
  #print("----------------------------------");
  fpcir[31]:=plot(Pcim[31],Srev[31]..0.99,-25..10,numpoints=300,
  color=blue):
> Srev[32]:=0.24:
> Sw_min[32,1]:=Sw_mini[31]:
> epsw[1]:=2:
  epso[1]:=2:
  b[0]:=1:
  a[0]:=1:
  for i from 1 to 10 do
  S_nw:=(S-Sw_min[32,i])/(1-Sw_min[32,i]):
  S_no:= (1-S-So_min[32,i])/(1-So_min[32,i]):
```

```
  Pcdr[32]:= cwd/S_nw^awd+cod/S_no^aod:
  Pcdr[32]:= unapply(Pcdr[32],S):
  So_min[32,i]:=solve(Pcim[31](Srev[32])=Pcdr[32](Srev[32]),So_min
  [32,i]):
  a[i]:=So_min[32,i]:
  S_nw:= (S-Sw_min[32,i+1])/(1-Sw_min[32,i+1]):
  S_no:= (1-S-So_min[32,i])/(1-So_min[32,i]):
  Pcdr[32]:= cwd/S_nw^awd+cod/S_no^aod:
  Pcdr[32]:= unapply(Pcdr[32],S):
  Sw_min[32,i+1]:= solve(Pcim[31](Srev[31])=Pcdr[32](Srev[31]),
  Sw_min[32,i+1]):
  b[i]:=Sw_min[32,i+1]:
  epso[i]:=abs(a[i]-a[i-1]):
  epsw[i]:=abs(b[i]-b[i-1]):
  eps[i]:=epsw[i]+epso[i]:
  #print(eps[i]);
  if (eps[i]<0.002)
  then
  #print(i);
  i:=11
  fi:
  Sw_min[32]:=Sw_min[32,i];
  So_min[32]:=So_min[32,i];
  #print(Sw_min = Sw_min[32]);
  #print(So_min = So_min[32]);
  od:
> fpcdr[32]:=plot(Pcdr[32],Srev[31]..Srev[32], color=green,labels=
  [Saturation ( ),CapillaryPressure (kPa)],thickness=1,
  labeldirections=[horizontal,vertical]):
  fpcir[31]:=plot(Pcim[31],Srev[31]..Srev[32], color=blue,labels=
  [Saturation ( ),CapillaryPressure (kPa)],thickness=1,
  labeldirections=[horizontal,vertical]):
> display(fpci1,fpci,fpcd,fpcd1,fpcD1,fpcir1,fpcdr2,fpcir[3],fpcdr
  [4],fpcir[5],fpcdr[6],fpcir[7],fpcdr[8],fpcir[9],fpcdr[10],fpcir
  [11],fpcdr[12],fpcir[13],fpcdr[14],fpcir[15],fpcdr[16],fpcir[17],
  fpcdr[18],fpcir[19],fpcdr[20],fpcir[21],fpcdr[22],fpcir[23],fpcdr
  [24],fpcir[25],fpcdr[26],fpcir[27],fpcdr[28],fpcir[29],fpcdr[30],
  fpcir[31],fpcdr[32]);
```


30
20
10
0
-10
-20
0.2
0.4
0.6
0.8
1
CapillaryPressure(kPa)
Saturation( )

A.3 Centrifuge Mode

```
> restart:
```

Centrifuge Experiment

```
> with(RealDomain):
  with(LinearAlgebra):
  with(ExcelTools):
> with(plots):
```

General Definitions for drainage and imbibition capillary pressure curves

```
> Swn:=(S-Swr)/(1-Swr):
  Son:=(1-S-Sor)/(1-Sor):
> pcpi:= cwi/Swn^awi:   pcmi:= coi/Son^aoi:
  pcpd:= cwd/Swn^awd:   pcmd:= cod/Son^aod:
> Pci:= unapply(pcpi+pcmi,S):
  Pcd:= unapply(pcpd+pcmd,S):
  PcD1:= unapply(pcpd,S):
```

Swn ... normalized water saturation, Son ... normalized oil saturation
pcpi,pcpd ...c apillary pressrue positive part of imbibition/drainage
pcmi,pcmd ... capillary pressrue minus part of imbibition/drainage
Pci ... capillary pressure imbibtion, Pcd ... capillary pressure drainage
PcD1 ... capillary pressure of primary drainage
cwi,cwd ... entry pressure for water (i for imbibition entry pre and d for drainage entry pre)
coi,cod ... entry pressure for oil (i for imbibition entry pre and d for drainage entry pre)
1/a .. .poresize distribution (w...water,o...oil)

Determine coi and cod

```
> coi:=solve(Pci(S1)=0,coi):
  cod:=solve(Pcd(S2)=0,cod):
```

S1 and S2 is the crossover points between respectively primary imbibition and secondary drainage and the "S-axis". Pc(S) is of course zero.

Lands Equation

```
> C:=1/Sor-1/(1-Swr):
```

C ... Land's trapping constant
Using Land's equation to determine asymptotes when imbibition starts from primary drainage curve .

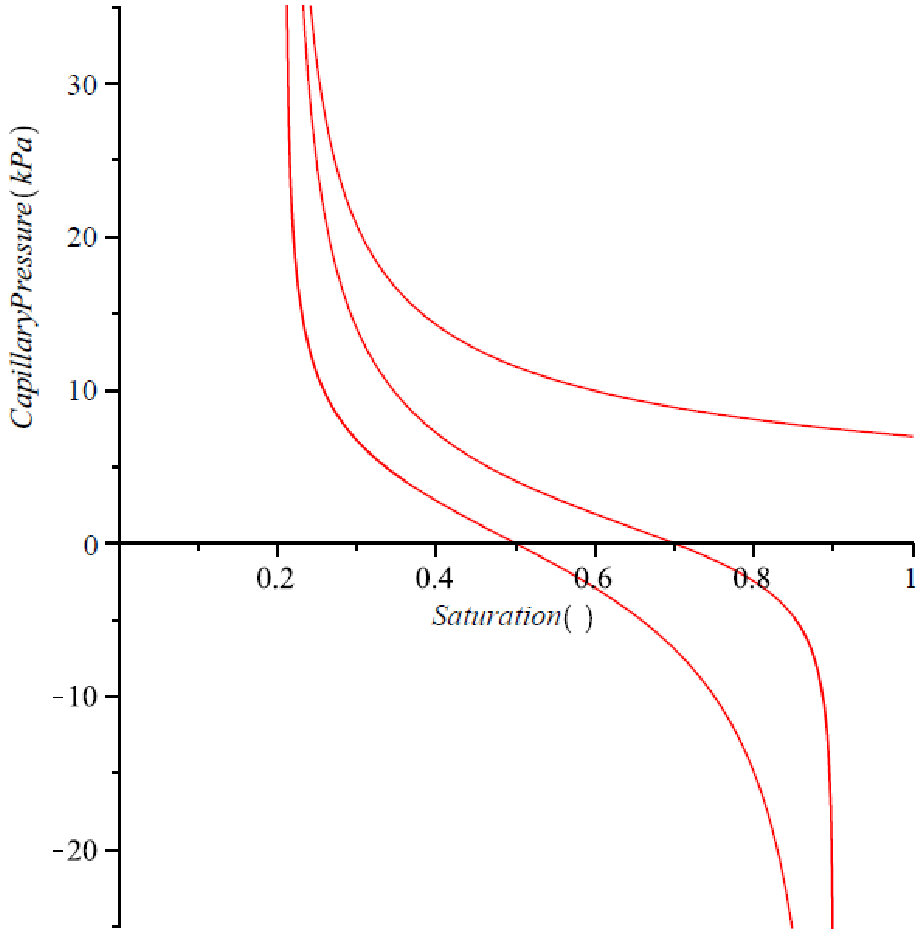
Defining Constants

```
> awi:=0.25:  aoi:=0.5:  cwi:=10:
  awd:=0.5:  aod:=0.25:  cwd:=7:
  Sor:=0.1:  Swr:=0.21:
  S1:=0.5:  S2:=0.7:
  coi:cod:
```

Plot of primary drainage and imbibiton bounding loop

```
> fpci:= plot(Pci,(Swr-0.05)..(Swr+0.15),0..35,numpoints=
  500,labels=[Saturation ( ),CapillaryPressure (kPa)],thickness=1,
  labeldirections=[horizontal,vertical]):
  fpci1:= plot(Pci,Swr..1,-25..25,numpoints=100,labels=[Saturation
  ( ),CapillaryPressure (kPa)],thickness=1,labeldirections=
  [horizontal,vertical]):
  fpcd:= plot(Pcd, Swr..1,-25..25,numpoints=100,labels=[Saturation
  ( ),CapillaryPressure (kPa)],thickness=1,labeldirections=
  [horizontal,vertical]):
  fpcd1:= plot(Pcd, (1-Sor-0.05)..(1-Sor+0.05),-25..25,numpoints=
  500,labels=[Saturation ( ),CapillaryPressure (kPa)],thickness=1,
  labeldirections=[horizontal,vertical]):
```

```
fpcD1:= plot(PcD1, 0..1,labels=[Saturation ( ),CapillaryPressure
(kPa)],thickness=1,labeldirections=[horizontal,vertical]):
> display(fpci1,fpci,fpcd,fpcd1,fpcD1);
```



Staring the centrifuge procedure

Input values, the values for r2 (a little different as in my case r2 should be the FWL and not the end of the core), r1 and dealta_rho are taken from the Hermansen et al (1991). Using r2 and r1 the different loctions (r) are evaluated, there should be as many as passible saturation reversals therefore the 54 points are chosen.

Defining parameters:

```
> r2:=0.0930:
  r1:=0.0446:
  r3:=0.069:
  rho_oct:=777:
  roh_wa:=1000:
  deltarho:=roh_wa-rho_oct:
```

Defining radius:

Locations at the core where the capillary pressure and staturation should be determined.

```
> for j from 1 to 54 do
  if j < 38 then
  r[j]:=r2-((r2-r1)/80)*j:
```

```
  #print(j = r[j]);
  else
  r[j]:=r3-((r3-r1)/16)*(j-38):
  #print(j = r[j]);
  fi:
  od:
```

Defining centrifuge speed:

Speeds at which the capillary pressure and saturation values should be determined at each position. In this case in 500 RPM steps.

```
> for j from 1 to 10 do
  omega[j]:=5000-500*(j-1):
  od:
> omega[11]:=100:
```

Calculation part Srev, Pcim, SorD and Sw_min at 5000RPM:

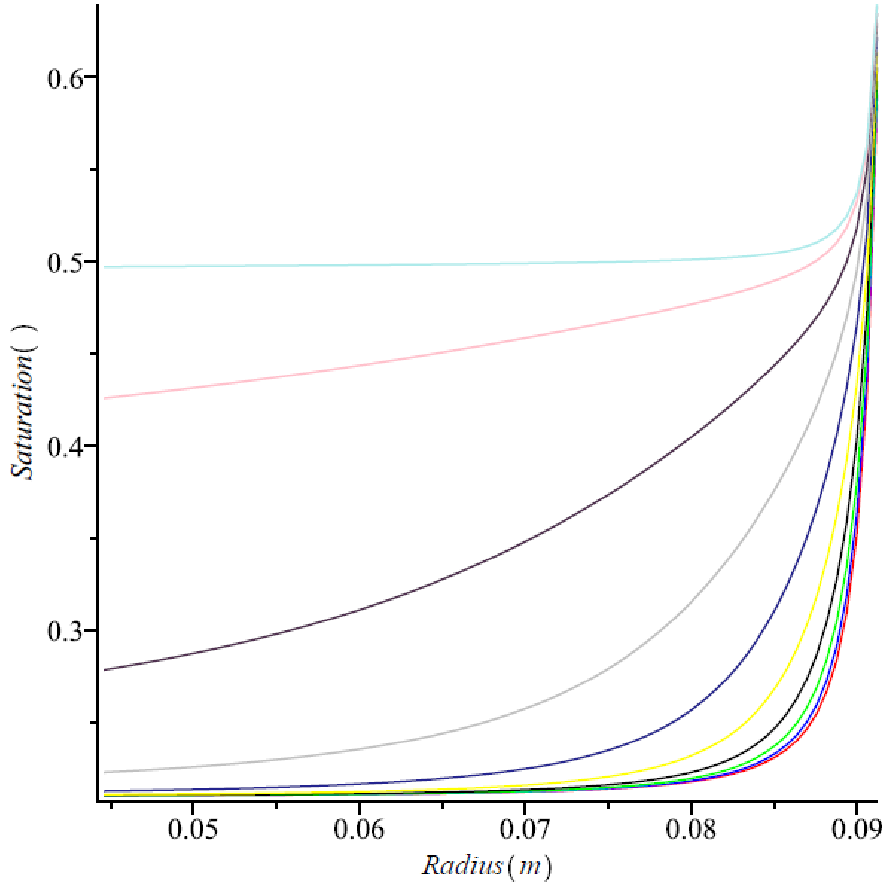
```
> for i from 3 to 54 do
  #print(r= r[i]);
  Pc[i]:=(((1/2)*(2*Pi*omega[1]/60)^2*deltarho*(r2^2-r[i]^2))/1000)
  ;
  Srev[i]:=fsolve(PcD1(S)=Pc[i],S);
  #print("Srev=", Srev[i]);
  SorD[i]:=1/(C+(1/(1-Srev[i])));
  S_nw[i]:=(S-Sw_min[i])/(1-Sw_min[i]);
  S_no[i]:=(1-S-SorD[i])/(1-SorD[i]);
  Pcim[i]:=cwi/S_nw[i]^awi+coi/S_no[i]^aoi:
  Pcim[i]:=unapply(Pcim[i],S);
  Sw_min[i]:=fsolve(Pcim[i](Srev[i])=Pc[i],Sw_min[i]);
  #print("Sw_min=", Sw_min[i]);
  #("--------------------------")
  od:
```

Calculation for the spontaneous imbibition capillary pressure curves for every point from 3 to 54 (1 and 2 the core is 100% saturated) at the predefined centrifuge speeds:

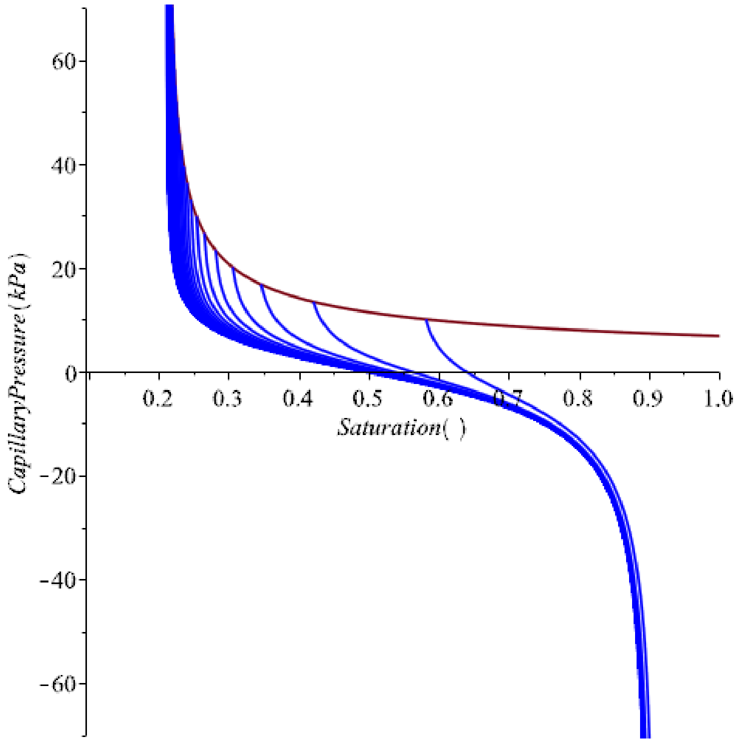
```
> for j from 2 to 11 do

  #print("j=", j);
  for i from 3 to 54 do
  #print("r=", r[i]);
  Pc[i]:=((1/2)*(2*Pi*omega[j]/60)^2*deltarho*(r2^2-r[i]^2))/1000;
  Pcim[i]:=cwi/S_nw[i]^awi+coi/S_no[i]^aoi:
  Pcim[i]:=unapply(Pcim[i],S);
  Sny[i]:=solve(Pcim[i](S)=Pc[i],S);
  #print("Sny=", Sny[i]);
  #print("-------------------------");
  fpcir[j,i]:=plot(Pcim[i],Srev[i]..0.99, color=blue,labels=
  [Saturation( ), CapillaryPressure (kPa)],labeldirections=
  [horizontal,vertical]):
  od:

  #print("________END j________", j);
  smst[j]:=array(3..54);

  col[j]:=array(1..16);
  col[2]:='red':
  col[3]:='blue':
  col[4]:='green':
  col[5]:='black':
```

```
col[6]:='yellow':
col[7]:='navy':
col[8]:='grey':
col[9]:='violet':
col[10]:='pink':
col[11]:='turquoise':

# loop to plot values
for i from 3 to 54 do
smst[j][i]:=r[i],Sny[i] od:
fsmsv[j]:=pointplot(smst[j],connect=true,color=col[j],labels=
[Radius(m),Saturation( )],labeldirections=[horizontal,vertical]):

#  prepare values to export to excel
A[j]:=Matrix(52,2,[r[3],Sny[3],r[4],Sny[4],r[5],Sny[5],r[6],Sny
[6],r[7],Sny[7],r[8],Sny[8],r[9],Sny[9],r[10],Sny[10],r[11],Sny
[11],r[12],Sny[12],r[13],Sny[13],r[14],Sny[14],r[15],Sny[15],r
[16],Sny[16],r[17],Sny[17],r[18],Sny[18],r[19],Sny[19],r[20],Sny
[20],r[21],Sny[21],r[22],Sny[22],r[23],Sny[23],r[24],Sny[24],r
[25],Sny[25],r[26],Sny[26],r[27],Sny[27],r[28],Sny[28],r[29],Sny
[29],r[30],Sny[30],r[31],Sny[31],r[32],Sny[32],r[33],Sny[33],r
[34],Sny[34],r[35],Sny[35],r[36],Sny[36],r[37],Sny[37],r[38],Sny
[38],r[39],Sny[39],r[40],Sny[40],r[41],Sny[41],r[42],Sny[42],r
[43],Sny[43],r[44],Sny[44],r[45],Sny[45],r[46],Sny[46],r[47],Sny
[47],r[48],Sny[48],r[49],Sny[49],r[50],Sny[50],r[51],Sny[51],r
[52],Sny[52],r[53],Sny[53],r[54],Sny[54]]);
od:
```

```
> display(fsmsv[2],fsmsv[3],fsmsv[4],fsmsv[5],fsmsv[6],fsmsv[7],
fsmsv[8],fsmsv[9],fsmsv[10],fsmsv[11]);
```



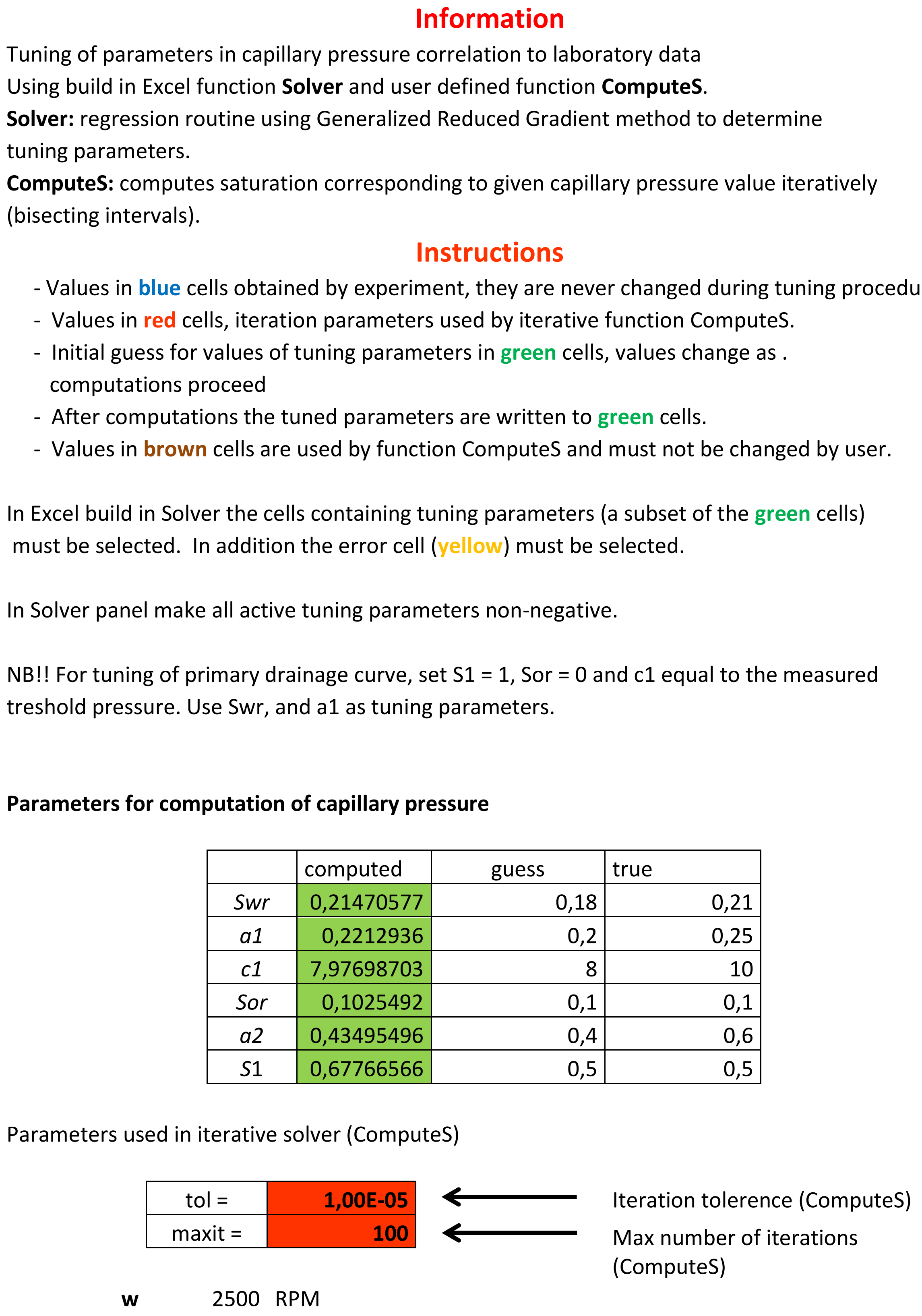
```
> display(fsmsv[2],fsmsv[3],fsmsv[4],fsmsv[5],fsmsv[6],fsmsv[7],
  fsmsv[8],fsmsv[9],fsmsv[10],fsmsv[11],view=[0.055..0.09,0.21.
  .0.25]):
> display(fpcD1,fpcd,fpcir[2,3],fpcir[2,4],fpcir[2,5],fpcir[2,6],
  fpcir[2,7],fpcir[2,8],fpcir[2,9],fpcir[2,10],fpcir[2,11],fpcir[2,
  12],fpcir[2,13],fpcir[2,14],fpcir[2,15],fpcir[2,16],fpcir[2,17],
  fpcir[2,18],fpcir[2,19],fpcir[2,20],fpcir[2,21],fpcir[2,22],fpcir
  [2,23],fpcir[2,24],fpcir[2,25],fpcir[2,26],fpcir[2,27],fpcir[2,
  28],fpcir[2,29],fpcir[2,30],fpcir[2,31],fpcir[2,32],fpcir[2,33],
  fpcir[2,34],fpcir[2,35],fpcir[2,36],fpcir[2,37],fpcir[2,38],fpcir
  [2,39],fpcir[2,40],fpcir[2,41],fpcir[2,42],fpcir[2,43],fpcir[2,
  44],fpcir[2,45],fpcir[2,46],fpcir[2,47],fpcir[2,48],fpcir[2,49],
  fpcir[2,50],fpcir[2,51],fpcir[2,52],fpcir[2,53],fpcir[2,54],view=
  [0.1..1,-70..70]);
```



```
> display(fpcD1,fpcd,fpcir[2,3],fpcir[2,4],fpcir[2,5],fpcir[2,6],
  fpcir[2,7],fpcir[2,8],fpcir[2,9],fpcir[2,10],fpcir[2,11],fpcir[2,
  12],fpcir[2,13],fpcir[2,14],fpcir[2,15],fpcir[2,16],fpcir[2,17],
  fpcir[2,18],fpcir[2,19],fpcir[2,20],fpcir[2,21],fpcir[2,22],fpcir
  [2,23],fpcir[2,24],fpcir[2,25],fpcir[2,26],fpcir[2,27],fpcir[2,
  28],fpcir[2,29],fpcir[2,30],fpcir[2,31],fpcir[2,32],fpcir[2,33],
  fpcir[2,34],fpcir[2,35],fpcir[2,36],fpcir[2,37],fpcir[2,38],fpcir
  [2,39],fpcir[2,40],fpcir[2,41],fpcir[2,42],fpcir[2,43],fpcir[2,
  44],fpcir[2,45],fpcir[2,46],fpcir[2,47],fpcir[2,48],fpcir[2,49],
  fpcir[2,50],fpcir[2,51],fpcir[2,52],fpcir[2,53],fpcir[2,54],view=
  [0.2..0.9,-10..70]):
> display(fpcD1,fpcd,fpcir[3,3],fpcir[3,4],fpcir[3,5],fpcir[3,6],
  fpcir[3,7],fpcir[3,8],fpcir[3,9],fpcir[3,10],fpcir[3,11],fpcir[3,
  12],fpcir[3,13],fpcir[3,14],fpcir[3,15],fpcir[3,16],fpcir[3,17],
  fpcir[3,18],fpcir[3,19],fpcir[3,20],fpcir[3,21],fpcir[3,22],fpcir
  [3,23],fpcir[3,24],fpcir[3,25],fpcir[3,26],fpcir[3,27],fpcir[3,
  28],fpcir[3,29],fpcir[3,30],fpcir[3,31],fpcir[3,32],fpcir[3,33],
  fpcir[3,34],fpcir[3,35],fpcir[3,36],fpcir[3,37],fpcir[3,38],fpcir
  [3,39],fpcir[3,40],fpcir[3,41],fpcir[3,42],fpcir[3,43],fpcir[3,
  44],fpcir[3,45],fpcir[3,46],fpcir[3,47],fpcir[3,48],fpcir[3,49],
  fpcir[3,50],fpcir[3,51],fpcir[3,52],fpcir[3,53],fpcir[3,54],view=
  [0.2..0.3,-10..70]):
> display(fpcD1,fpcd,fpcir[4,3],fpcir[4,4],fpcir[4,5],fpcir[4,6],
  fpcir[4,7],fpcir[4,8],fpcir[4,9],fpcir[4,10],fpcir[4,11],fpcir[4,
  12],fpcir[4,13],fpcir[4,14],fpcir[4,15],fpcir[4,16],fpcir[4,17],
  fpcir[4,18],fpcir[4,19],fpcir[4,20],fpcir[4,21],fpcir[4,22],fpcir
  [4,23],fpcir[4,24],fpcir[4,25],fpcir[4,26],fpcir[4,27],fpcir[4,
  28],fpcir[4,29],fpcir[4,30],fpcir[4,31],fpcir[4,32],fpcir[4,33],
  fpcir[4,34],fpcir[4,35],fpcir[4,36],fpcir[4,37],fpcir[4,38],fpcir
  [4,39],fpcir[4,40],fpcir[4,41],fpcir[4,42],fpcir[4,43],fpcir[4,
  44],fpcir[4,45],fpcir[4,46],fpcir[4,47],fpcir[4,48],fpcir[4,49],
  fpcir[4,50],fpcir[4,51],fpcir[4,52],fpcir[4,53],fpcir[4,54],view=
  [0.2..0.22,-10..200]):
> Export(A[2]);
> Export(A[3]);
> Export(A[4]);
> Export(A[5]);
> Export(A[6]);
> Export(A[7]);
> Export(A[8]);
> Export(A[9]);
> Export(A[10]);
```

A.4 Excel Tool

This part of the appendix should show how the Excel tool is build (the alignment was slightly changed for the Appendix).



Information

Tuning of parameters in capillary pressure correlation to laboratory data
Using build in Excel function **Solver** and user defined function **ComputeS**.
Solver: regression routine using Generalized Reduced Gradient method to determine tuning parameters.
ComputeS: computes saturation corresponding to given capillary pressure value iteratively (bisecting intervals).

Instructions

- Values in **blue** cells obtained by experiment, they are never changed during tuning procedue.
- Values in **red** cells, iteration parameters used by iterative function ComputeS.
- Initial guess for values of tuning parameters in **green** cells, values change as . computations proceed
- After computations the tuned parameters are written to **green** cells.
- Values in **brown** cells are used by function ComputeS and must not be changed by user.

In Excel build in Solver the cells containing tuning parameters (a subset of the **green** cells) must be selected. In addition the error cell (**yellow**) must be selected.

In Solver panel make all active tuning parameters non-negative.

NB!! For tuning of primary drainage curve, set S1 = 1, Sor = 0 and c1 equal to the measured treshold pressure. Use Swr, and a1 as tuning parameters.

Parameters for computation of capillary pressure

	computed	guess	true
Swr	0,21470577	0,18	0,21
a1	0,2212936	0,2	0,25
c1	7,97698703	8	10
Sor	0,1025492	0,1	0,1
a2	0,43495496	0,4	0,6
S1	0,67766566	0,5	0,5

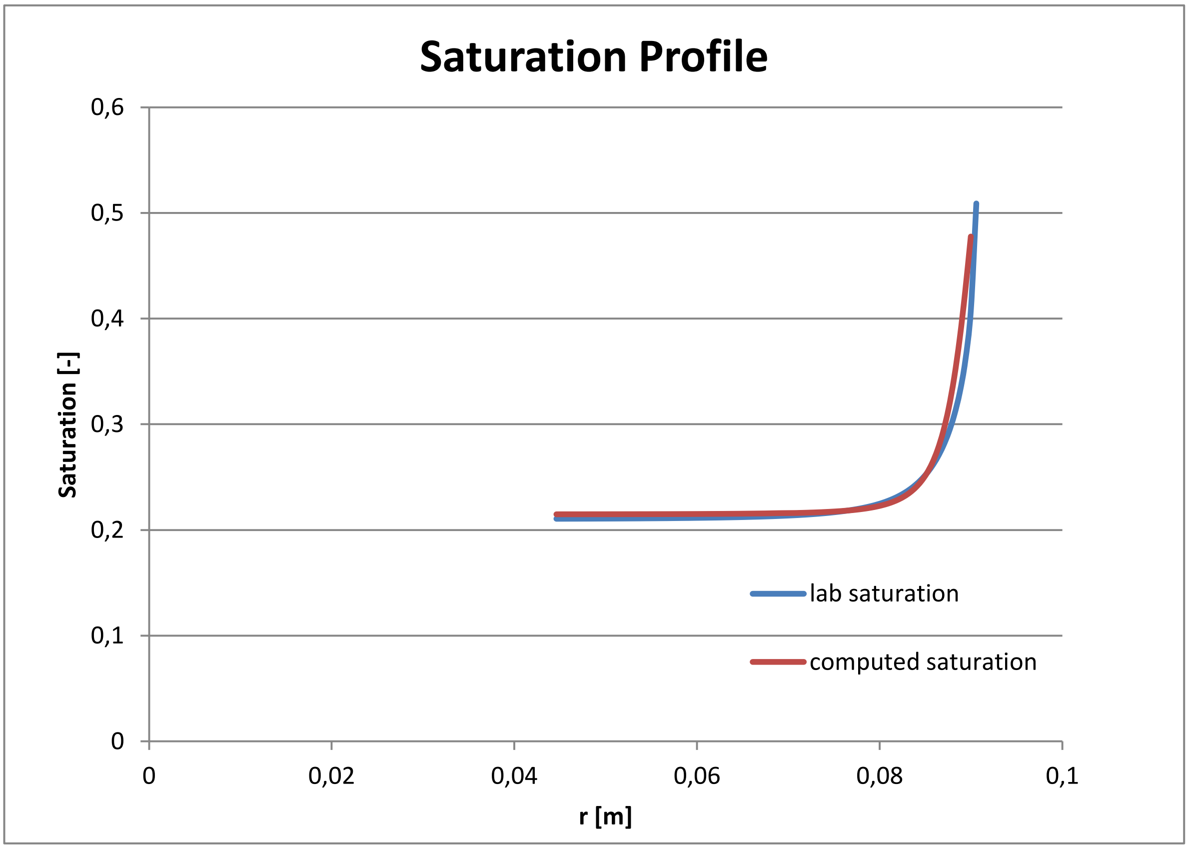
Parameters used in iterative solver (ComputeS)

tol =	**1,00E-05**	← Iteration tolerence (ComputeS)
maxit =	**100**	← Max number of iterations (ComputeS)

w 2500 RPM

r	Input Pc		Input saturation	Computed S	sqr of diff	Error
0,09058	3,3951022	**3,3951022**	0,50895345	**0,477676058**	0,00097828	0,00272226
0,089975	4,22989177	**4,22989177**	0,407355842	**0,426149311**	0,00035319	
0,08937	5,05908695	**5,05908695**	0,359838886	**0,381198839**	0,00045625	
0,088765	5,88268774	**5,88268774**	0,330039266	**0,344314398**	0,00020378	
0,08816	6,70069413	**6,70069413**	0,308660761	**0,315284373**	4,3872E-05	
0,087555	7,51310615	**7,51310615**	0,292298906	**0,292979079**	4,6264E-07	
0,08695	8,31992375	**8,31992375**	0,279352421	**0,276021078**	1,1098E-05	
0,086345	9,12114697	**9,12114697**	0,268911949	**0,263151599**	3,3182E-05	
0,08574	9,91677579	**9,91677579**	0,260388182	**0,253348879**	4,9552E-05	
0,085135	10,7068102	**10,7068102**	0,253365853	**0,245831415**	5,6768E-05	
0,08453	11,4912503	**11,4912503**	0,247537359	**0,240017922**	5,6542E-05	
0,083925	12,2700959	**12,2700959**	0,24266803	**0,23548078**	5,1657E-05	
0,08332	13,0433472	**13,0433472**	0,238575564	**0,231906721**	4,4473E-05	
0,082715	13,8110041	**13,8110041**	0,235116624	**0,229065412**	3,6617E-05	
0,08211	14,5730666	**14,5730666**	0,232177507	**0,226786423**	2,9064E-05	
0,081505	15,3295347	**15,3295347**	0,229667369	**0,22494295**	2,232E-05	
0,0809	16,0804083	**16,0804083**	0,227513169	**0,223439644**	1,6594E-05	
0,080295	16,8256876	**16,8256876**	0,225655847	**0,222204415**	1,1912E-05	
0,07969	17,5653726	**17,5653726**	0,224047383	**0,221182142**	8,2096E-06	
0,079085	18,299463	**18,299463**	0,222648533	**0,22033036**	5,3739E-06	
0,07848	19,0279594	**19,0279594**	0,221427058	**0,219616126**	3,2795E-06	
0,077875	19,7508611	**19,7508611**	0,220356352	**0,219013599**	1,803E-06	
0,07727	20,4681685	**20,4681685**	0,219414346	**0,218502452**	8,3155E-07	
0,076665	21,1798811	**21,1798811**	0,218582657	**0,218066499**	2,6642E-07	
0,07606	21,8859996	**21,8859996**	0,2178459	**0,217692799**	2,344E-08	
0,075455	22,5865242	**22,5865242**	0,217191147	**0,217370949**	3,2329E-08	
0,07485	23,2814541	**23,2814541**	0,216607488	**0,217092495**	2,3523E-07	
0,074245	23,9707889	**23,9707889**	0,216085679	**0,216850565**	5,8505E-07	
0,07364	24,6545305	**24,6545305**	0,21561786	**0,216639512**	1,0438E-06	
0,073035	25,332676	**25,332676**	0,21519732	**0,216454686**	1,581E-06	
0,07243	26,0052295	**26,0052295**	0,214818311	**0,216292243**	2,1725E-06	
0,071825	26,6721869	**26,6721869**	0,214475892	**0,216148974**	2,7992E-06	
0,07122	27,3335497	**27,3335497**	0,214165801	**0,216022197**	3,4462E-06	
0,070615	27,98932	**27,98932**	0,213884352	**0,215909663**	4,1019E-06	
0,069	29,7124438	**29,7124438**	0,213250219	**0,215665426**	5,8332E-06	
0,067475	31,3029485	**31,3029485**	0,212775749	**0,215491726**	7,3765E-06	
0,06595	32,857909	**32,857909**	0,212392658	**0,215357643**	8,7911E-06	
0,064425	34,3773241	**34,3773241**	0,212080064	**0,215252662**	1,0065E-05	
0,0629	35,8611934	**35,8611934**	0,211822524	**0,215169405**	1,1202E-05	
0,061375	37,3095196	**37,3095196**	0,211608463	**0,215102603**	1,2209E-05	
0,05985	38,7223	**38,7223**	0,211429093	**0,21504843**	1,31E-05	
0,058325	40,0995309	**40,0995309**	0,21127767	**0,21500407**	1,3886E-05	
0,0568	41,4412191	**41,4412191**	0,211148959	**0,21496742**	1,4581E-05	

0,055275	42,7473618	**42,7473618**	0,211038859	**0,214936891**	1,5195E-05
0,05375	44,0179537	**44,0179537**	0,210944124	**0,214911267**	1,5738E-05
0,052225	45,2530096	**45,2530096**	0,210862168	**0,21488961**	1,622E-05
0,0507	46,4525145	**46,4525145**	0,21079091	**0,214871186**	1,6649E-05
0,049175	47,616475	**47,616475**	0,210728666	**0,21485542**	1,703E-05
0,04765	48,7448859	**48,7448859**	0,210674061	**0,214841854**	1,737E-05
0,046125	49,837769	**49,837769**	0,210625967	**0,21483012**	1,7675E-05
0,0446	50,8950716	**50,8950716**	0,210583452	**0,214819924**	1,7948E-05



A.5 Macro – ComputeS

The Macro to evaluate the saturation values in Excel is introduced:

```
Function ComputeS(Pc As Double) As Double
'Compute saturation S given value Pc of capillary pressure
'Iterative method: bisecting interval

Dim S As Double
Dim i As Integer
Dim Err As Double

'Input Parameters for computing Pc specified in Data sheet
  Swr = Worksheets("Data").Cells(4, 3).Value
  a1 = Worksheets("Data").Cells(5, 3).Value
  c1 = Worksheets("Data").Cells(6, 3).Value
  Sor = Worksheets("Data").Cells(7, 3).Value
  a2 = Worksheets("Data").Cells(8, 3).Value
  S1 = Worksheets("Data").Cells(9, 3).Value

  S11 = (S1 + Sor - 1) / (Sor - 1)
  S22 = (Swr - S1) / (Swr - 1)

  c2 = -c1 * (S11 ^ a2) * (S22) ^ (-a1)

'Parameters for iteration procedure (bisecting interval)
  tol = Worksheets("Data").Cells(13, 3).Value
  maxit = Worksheets("Data").Cells(14, 3).Value

'Start values for iterations
  Smin = Swr
  Smax = 1 - Sor

'Start iterations
  i = 0
  Do While i < maxit
  i = i + 1

  S = 0.5 * (Smax + Smin)

  S1 = (S - Swr) / (1 - Swr)
  S2 = (1 - S - Sor) / (1 - Sor)

  ComputedPc = c1 * S1 ^ (-a1) + c2 * S2 ^ (-a2)

  Err = Pc - ComputedPc
```

```
    If Err < 0 Then
      DErr = -Err
    Else
      DErr = Err
    End If

    If DErr > tol Then

      If Err < 0 Then
        Smin = S
      Else
        Smax = S
      End If

    Else

      ComputeS = S
      Exit Do

    End If

  Loop

'  If i > maxit - 1 Then
'    MsgBox "NB!! no convergence"
'  End If

End Function
```