

# Geometry and Kinematics of a Convertible's Rear Side Window

Conference on Geometry - Theory and Applications

Vorau, 2011 - 06 - 23

The task

The basic idea

Step 1: A substitute window surface

Step 2: A suitable window motion

Step 3: Intermediate window surfaces

The task

# The Task

## Rear Side Windows of Convertibles



The task

# The Task

## Rear Side Windows of Convertibles



The task

# The Task

## Rear Side Windows of Convertibles



The task

# The Task

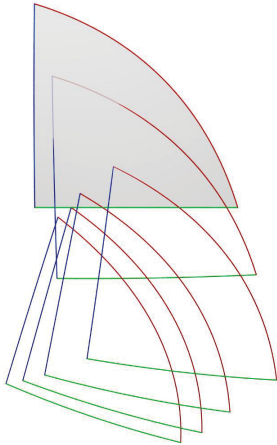
## Rear Side Windows of Convertibles



The Task

**Input Data:**

The Task

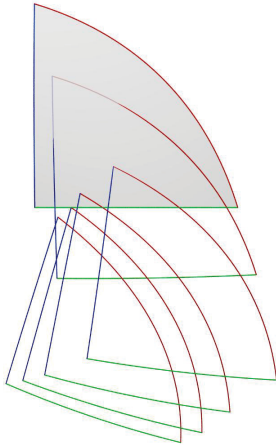


**Input Data:**

a surface  $S$  (rear side window suggested by the stylist)



### The Task

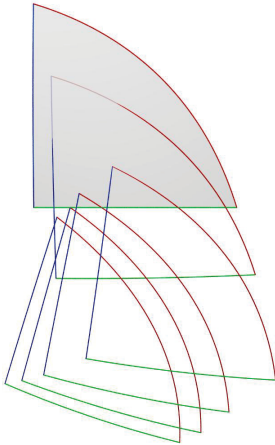


### Input Data:

a surface  $S$  (rear side window suggested by the stylist)

a couple of single window sheet positions

The Task



**Input Data:**

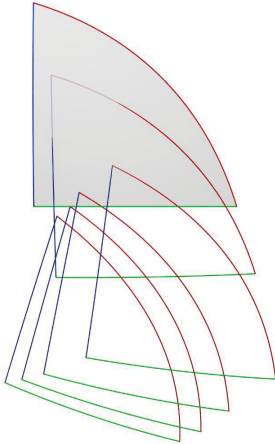
a surface  $S$  (rear side window suggested by the stylist)

a couple of single window sheet positions

the boundary curves:

- ▶ front boundary curve  $b$
- ▶ roofline curve  $r$
- ▶ daylight curve  $d$

## The Task



### Input Data:

a surface  $S$  (rear side window suggested by the stylist)

a couple of single window sheet positions

the boundary curves:

- ▶ front boundary curve  $b$
- ▶ roofline curve  $r$
- ▶ daylight curve  $d$

### Wanted:

a motion of  $S$  that

- ▶ interpolates the given positions and
- ▶ minimizes the stress on the sealing

The Task

**Strategy:**

The Task

## Strategy:

**Step 1:** Construction of a substitute window surface  $S_d$

The Task

**Strategy:**

**Step 1:** Construction of a substitute window surface  $S_d$

**Step 2:** Computation of a suitable window motion  $\mu$  by means of  $S_d$

**Strategy:**

**Step 1:** Construction of a substitute window surface  $S_d$

**Step 2:** Computation of a suitable window motion  $\mu$  by means of  $S_d$

**Step 3:** Construction of a set of intermediate window surfaces  $S_\beta$

## The basic idea



The basic idea

In order to find the appropriate window motion  
we first construct the 'perfect window surface'  $S_d$ .

The basic idea

In order to find the appropriate window motion  
we first construct the 'perfect window surface'  $S_d$ .

Which is the perfect window surface  $S_d$ ?

The basic idea

In order to find the appropriate window motion  
we first construct the 'perfect window surface'  $S_d$ .

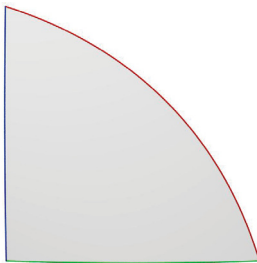
Which is the perfect window surface  $S_d$ ?

$S_d$  is surface of motion

The basic idea

In order to find the appropriate window motion  
we first construct the 'perfect window surface'  $S_d$ .

Which is the perfect window surface  $S_d$ ?

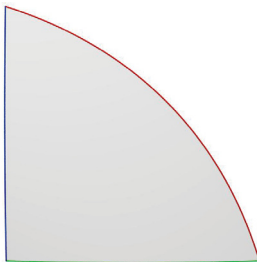


$S_d$  is surface of motion  
the generating curve of  $S_d$ : daylight curve  $d$

The basic idea

In order to find the appropriate window motion  
we first construct the 'perfect window surface'  $S_d$ .

Which is the perfect window surface  $S_d$ ?



$S_d$  is surface of motion

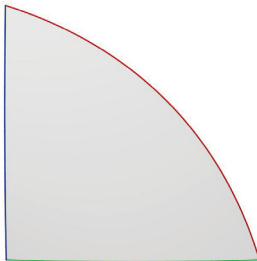
the generating curve of  $S_d$ : daylight curve  $d$

$S_d$  maintains the roofline curve  $r$

The basic idea

In order to find the appropriate window motion  
we first construct the 'perfect window surface'  $S_d$ .

Which is the perfect window surface  $S_d$ ?



$S_d$  is surface of motion

the generating curve of  $S_d$ : daylight curve  $d$

$S_d$  maintains the roofline curve  $r$

$S_d$  maintains the front boundary curve  $b$

Step 1: A substitute window surface

Step 1: A substitute window surface

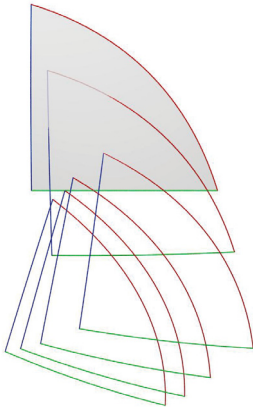
The inverse problem:



Step 1: A substitute window surface

The inverse problem:

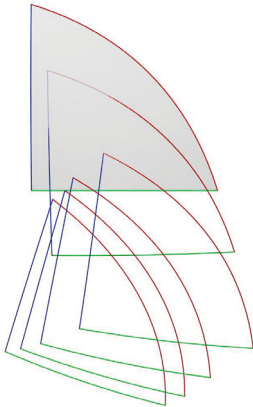
Sliding the window  
through the daylight  
curve (motion  $\mu$ ) ...



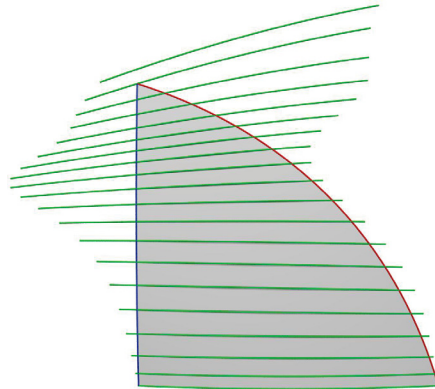
Step 1: A substitute window surface

The inverse problem:

Sliding the window through the daylight curve (motion  $\mu$ ) ...



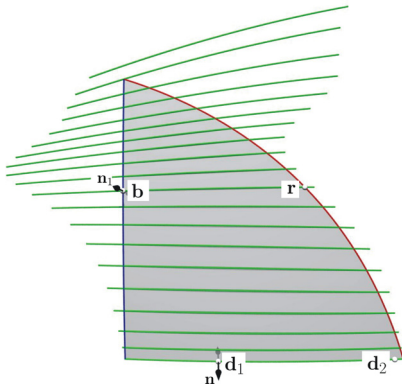
amounts to slipping the daylight curve across the window (inverse motion  $\mu^*$ ).



Step 1: A substitute window surface

Additional Constraints:

Step 1: A substitute window surface



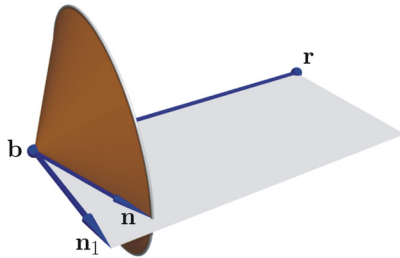
Additional Constraints:

Each given window sheet position suggests a certain position of the daylight curve during the motion  $\mu^*$ .

Step 1: A substitute window surface

Fine tuning the positions  
recognizes the tangent planes  
along the front boundary curve.

Step 1: A substitute window surface



Fine tuning the positions recognizes the tangent planes along the front boundary curve.

Minimizing the angle between the two normals: The chord and the two normals have to be coplanar.

## Step 2: A suitable window motion

Step 2: A suitable window motion

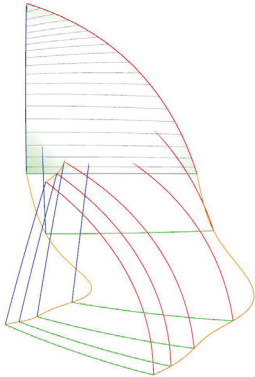


Step 2: A suitable window motion

An interpolation problem in the space of  
Euclidean Motions:

Result: A smooth spatial motion  $\mu^*$   
recognizing all conditions.

Step 2: A suitable window motion



An interpolation problem in the space of Euclidean Motions:

Result: A smooth spatial motion  $\mu^*$  recognizing all conditions.

The inverse motion  $\mu$ : A smooth spatial motion assuming each of the given window positions.

Step 2: A suitable window motion

The window motion  $\mu$  will move  $S_d$  perfectly.  
Zero stress on the sealing.



### Step 3: Intermediate window surfaces

Step 3: Intermediate window surfaces

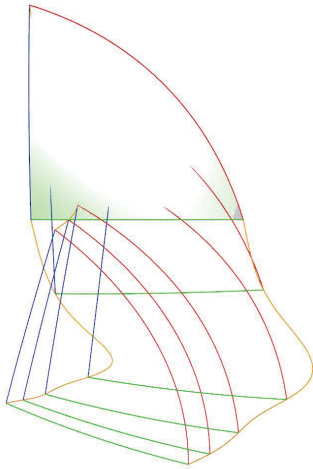
What if the given window surface  $S$  stresses the sealings beyond their prescribed limits?

Step 3: Intermediate window surfaces

What if the given window surface  $S$  stresses the sealings beyond their prescribed limits?

We stick to the constructed window motion  $\mu$ .

Step 3: Intermediate window surfaces



What if the given window surface  $S$  stresses the sealings beyond their prescribed limits?

We stick to the constructed window motion  $\mu$ .

$\mu$  will move  $S_d$  without any stress to the daylight sealing.





Step 3: Intermediate window surfaces

$$S_\beta \dots \mathbf{x}_\beta(u, v, w) := \mathbf{x}(u, v, w) + \beta(u, v, w) \cdot (\mathbf{x}_d(u, v, w) - \mathbf{x}(u, v, w))$$

Step 3: Intermediate window surfaces

$$S_\beta \dots \mathbf{x}_\beta(u, v, w) := \mathbf{x}(u, v, w) + \beta(u, v, w) \cdot (\mathbf{x}_d(u, v, w) - \mathbf{x}(u, v, w))$$

$$\mathcal{D} := \{(u, v, w) | u, v, w \in [0, 1], u + v + w = 1\}$$

Step 3: Intermediate window surfaces

$$S_\beta \dots \mathbf{x}_\beta(u, v, w) := \mathbf{x}(u, v, w) + \beta(u, v, w) \cdot (\mathbf{x}_d(u, v, w) - \mathbf{x}(u, v, w))$$

$$\mathcal{D} := \{(u, v, w) \mid u, v, w \in [0, 1], u + v + w = 1\}$$

$$\beta(u, v, w) := c u^k v^l w^m$$

Step 3: Intermediate window surfaces

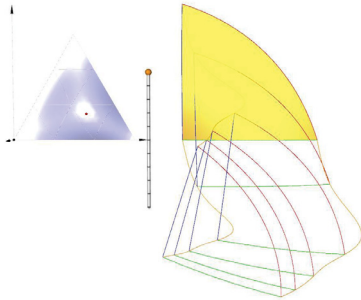
$$S_\beta \dots \mathbf{x}_\beta(u, v, w) := \mathbf{x}(u, v, w) + \beta(u, v, w) \cdot (\mathbf{x}_d(u, v, w) - \mathbf{x}(u, v, w))$$

$$\mathcal{D} := \{(u, v, w) \mid u, v, w \in [0, 1], u + v + w = 1\}$$

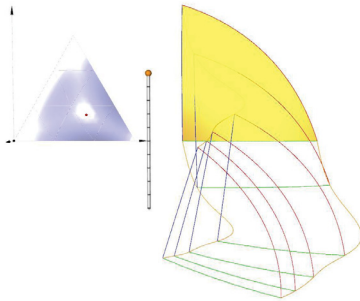
$$\beta(u, v, w) := c u^k v^l w^m$$

$$c := \frac{(k+l+m)^{k+l+m}}{k^k l^l m^m} \cdot b_0$$

Step 3: Intermediate window surfaces



Step 3: Intermediate window surfaces



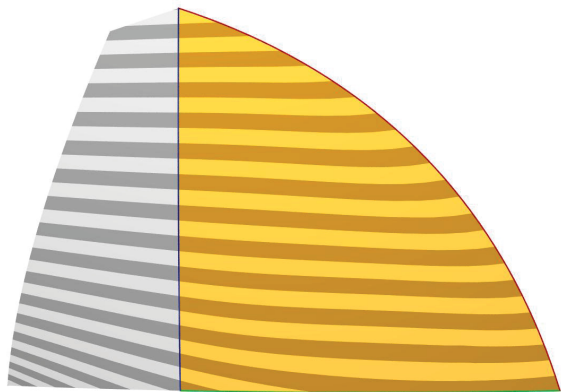
The intermediate surfaces  $S_\beta$  will enable the engineer to control











Thanks for your attention!